

Studentnumber: _____

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INSTITUTT FOR FYSIKK

Contact during the exam:
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EXAM: TFY4245 FASTSTOFF-FYSIKK VK

Monday 19. May 2008
Tid: kl 09.00-13.00

Allowed exam material: Alternative C
Standard pocket calculator
Rottman: Mathematical Formula (all language editions)
Barnett og Cronin: Mathematical Formula

The exam consists of:

1. The first page (the present page) which must be delivered with answers to the multiple choice questions.
2. 3 "normal" Problems 1, 2 and 3 (Appendix A)
3. One set of multiple choice questions, Problem 4 (Appendix B)

The three "normal" problems count altogether 50%, and the multiple choice questions count altogether 50%. Only ONE of the alternatives A-D must be marked for each of the 20 multiple choice questions. Correct answer gives one point, wrong answer gives zero points.

Answers to the multiple choice questions in Appendix B:

Question	1	2	3	4	5	6	7	8	9	10	11	12
Answer												

Question	13	14	15	16	17	18	19	20
Answer								

Problem 1. Linear response:**1a)**

Consider a physical system that is forced out of equilibrium by applying a time (t) dependent external force, $\sigma(t)$, to it.

For linear response, the response field perturbation of the system is:

$$\gamma(t) = \alpha \sigma(t) \quad (1)$$

where α is a linear response function which is characteristic of the specific system under consideration.

Assuming that the system returns to equilibrium, $\gamma(t = \infty)$, at a rate, $d\gamma(t)/dt$, which increases proportionally with the magnitude of the perturbation, $\gamma(t)$, we may write:

$$d\gamma(t)/dt = (\gamma(t = \infty) - \gamma(t))/\tau \quad (2)$$

where the system characteristic time, τ , is the constant of proportionality between the rate and the perturbation.

τ contains information about the dynamics of physical processes in the system that couple to the external field σ .

Equation (1) represents a simple relaxation process (Debye relaxation).

Assume, that we apply a time dependent periodic force to the system, such that:

$$\sigma(t) = \sigma_0 \exp(-i\omega t) \quad (3)$$

where ω is the applied frequency, and σ_0 is the force amplitude.

The resulting linear response can be written:

$$\gamma(t) = \alpha(\omega) \sigma(t) = \gamma_0 \exp(-i\omega t + i\delta(\omega)) \quad (4)$$

where $\delta(\omega)$ is the phase difference between the force and the response, and γ_0 is the response amplitude.

- Use the above equations in order to derive an expression for the complex frequency dependent linear response function

$$\alpha(\omega) = \exp(i\delta(\omega)) = \gamma(t)/\sigma(t) = \alpha'(\omega) + i\alpha''(\omega) \quad (5)$$

for the case of a simple relaxation process like the one in Equation (2). In addition sketch both $\alpha'(\omega) = \text{Re}(\alpha(\omega))$ and $\alpha''(\omega) = \text{Im}(\alpha(\omega))$ for this case. ($i = \sqrt{-1}$)

1b)

During a cyclic experiment like the one described in a), the power per unit volume provided by the external force may be written:

$$dw/dt = \text{Re}\alpha(t)d(\text{Re}\gamma(t))/dt \quad (6)$$

where Re means Real part.

- Show that Equation (6) may be rewritten in the form

$$dw/dt = -(1/2) \sigma_0^2 \omega \alpha'(\omega) \sin(2\omega t) + \sigma_0^2 \omega \alpha''(\omega) \cos^2(\omega t) \quad (7)$$

and discuss the meaning of the two terms in Equation (7).

- Use Equation (7) to write down an expression for the time-averaged power take-up by the system, and discuss the meaning of the result.

Note typo in Eq. (6), should say $dw/dt = \text{Re}\sigma(t)d(\text{Re}\gamma(t))/dt$

1c)

- For general linear response functions $\alpha(\omega)$, (i.e. not necessarily simple relaxation), derive the following Kramers-Kronig relations between the real and imaginary parts respectively:

$$\alpha'(\omega_0) = (1/\pi)P\int[\alpha''(\omega)/(\omega-\omega_0)]d\omega \quad (8)$$

$$\alpha''(\omega_0) = -(1/\pi)P\int[\alpha'(\omega)/(\omega-\omega_0)]d\omega \quad (9)$$

where the integral limits are from $-\infty$ to $+\infty$, and the principal value, P is defined as

$$P\int[\alpha(\omega)/(\omega-\omega_0)]d\omega = \lim_{\delta \rightarrow 0} \left\{ \int_1 [\alpha(\omega)/(\omega-\omega_0)]d\omega + \int_2 [\alpha(\omega)/(\omega-\omega_0)]d\omega \right\} \quad (10)$$

where the limits of integral 1 ($\int_1$) are between $-\infty$ and $\omega_0-\delta$, and those of integral 2 ($\int_2$) are between $\omega_0+\delta$ and ∞ .

- Discuss the physical contents of the Kramers-Kronig relations.

Problem 2. Nematics and the Isotropic-Nematic transition:**2a)**

- What is the general meaning of, and the general definition of, an order parameter in the context of phase transitions?
- Discuss what is meant by a nematic phase.
- Show that the order parameter for nematic ordering most suitably is chosen to be

$$S_2 = \langle (3\cos^2\theta - 1)/2 \rangle \quad (10)$$
- and discuss the meaning of the θ -angle, as well as the meaning of the averaging denoted by $\langle \rangle$ for this case.

2b)

Landau theory is a thermodynamic theory describing phase transitions, and this theory considers the Helmholtz free energy $f(\eta, T)$, where η is the order parameter, and T is the temperature: Landau theory assumes that near the phase transition temperature T_C , it is possible to expand the Helmholtz free energy in powers of η , thus

$$f(\eta, T) = f_0 + c_2\eta^2 + d_3\eta^3 + c_4\eta^4 + c_5\eta^5 + c_6\eta^6 + \dots + c_n\eta^n + \dots \quad (11)$$

Assume that $c_2(T) = b(T - T_C)$, where b is a positive constant, and also assume that $c_3, c_4, c_5, c_6, \dots, c_n, \dots$ are constants independent of T .

- Consider the case when $d_3 = -c_3$ is negative, c_4 is positive, and $c_n = 0$ for $n > 4$, and argue why such a non-zero c_3 is necessary in order to describe the isotropic-nematic transition, i.e. we are considering the Landau-de Gennes model for the nematic transition which then is:

$$f(S_2, T) = f_0 + c_2S_2^2 - c_3S_2^3 + c_4S_2^4 \quad (12)$$

- Use Equation (12) to show that the isotropic to nematic transition in this model is a first order phase transition, and that at the transition temperature T_{ni} , $S_2(T_{ni}) = C_3/(2C_4)$.
- Use Equation (12) to calculate $S_2(T)$ for $T < T_{ni}$.

2c)

The Maier-Saupe theory for nematics introduces a molecular field which acts to orient the rod-like molecules of standard nematics. This molecular field has the form

$$u(\theta) = -u_0 S_2 ((3\cos^2\theta - 1)/2) \quad (13)$$

- Discuss the meaning of the molecular field and its terms as given in Equation (13)

Boltzmann statistics gives the orientation distribution function $w(\theta, \phi)$ for this case, i.e.

$$w(\theta, \phi) = \exp(u(\theta)/k_B T) / Z \quad (14)$$

where k_B is Boltzmann's constant, T is the temperature, and the normalization factor Z is the partition function.

- Use Equation (14) to show how it can be calculated from this model that $S_2(T_{ni}) = 0.44$
- Discuss the conceptual difference between the Landau-de Gennes theory introduced in b) and the present Maier-Saupe approach.

Problem 3. Superconductivity:**3a)**

- Sketch and discuss magnetization curves (i.e. magnetization $\mathbf{M}$ vs magnetic field $\mathbf{H}$) for type I and type II superconductors respectively.

3b)

Using Ginzburg-Landau wave mechanics, the superconducting current density (for Cooper pairs) may be written

$$\mathbf{j} = -2e|\psi|^2(\hbar\nabla\theta(\mathbf{r})/2\pi + 2e\mathbf{A})/2m_e \quad (15)$$

where $\psi(\mathbf{r}) = |\psi(\mathbf{r})|\exp(i\theta(\mathbf{r}))$ is the wavefunction with phase $\theta(\mathbf{r})$, $2e$ is the Cooper pair charge, $2m_e$ is the Cooper pair mass, and $\mathbf{A}$ is the vector potential, related to the magnetic field through $\nabla \times \mathbf{A} = \mathbf{B}$.

- Show how you can combine Equation (15) with the Maxwell equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$, in order to derive the London equation

$$\nabla^2 \mathbf{B} = \mathbf{B} \lambda_L^{-2} \quad (16)$$

where λ_L is the London penetration depth, defined as

$$\lambda_L^{-2} = \mu_0 \rho_S (-2e)^2 / (2m_e) \quad (17)$$

and the Cooper pair density $\rho_S = |\psi|^2$.

- Discuss and sketch the meaning of λ_L .

3c)

The Ginzburg-Landau thermodynamic theory for superconductivity assumes Equation (11) for superconductivity with $|\psi|$ as the orderparameter, and b and c_4 positive, $c_3 = 0$ and $c_n = 0$ for $n > 4$.

- Use these assumptions to calculate the equilibrium value $\rho_S = |\psi|_{eq}^2$ for $T < T_C$.

3d)

Type II superconductors may involve spatial variations of ρ_S throughout the sample, i.e. Equation (11) may not be applicable, since it involves a spatially homogenous order parameter.

In the Ginzburg-Landau-Abrikosov theory for superconductors, Equation (11) is replaced by this expression for the free energy:

$$g(|\psi|, T) = g_0 + b(T - T_C)|\psi|^2 + c_4|\psi|^4 + (\hbar/8\pi m_e)^2 |\nabla \psi|^2 + (\text{magnetic field dependent terms}) \quad (18)$$

- Use dimensional analysis together with Equation (18) to show that there must be a length describing spatial variations of $\rho_S = |\psi|^2$ in the sample, and that this length, ξ_G , has the form

$$\xi_G \propto (T_C - T)^{-1/2} \text{ for } T < T_C \quad (19)$$

ξ_G is the coherence length.

- Discuss the meaning of ξ_G in relation to λ_L derived in b) above, in the context of type II superconductors.

Note typo in Eq.(15), should say $\mathbf{j} = -2e|\psi|^2(\hbar\nabla\theta(\mathbf{r})/2\pi + 2e\mathbf{A})/2m_e$

Problem 4. Multiple choice questions:

1. The pair-distribution function $g_2(r)$ for an isotropic atomic amorphous solid or a liquid display a series of maxima and minima vs r . The minima signify:
 - a) Preferred positions of shells of atoms around a chosen reference atom
 - b) Intensity maxima of scattered x-rays from the liquid, where r is the sample to detector distance
 - c) The distribution of a liquid subjected to an electric field
 - d) None of the above
2. Bragg scattering peaks from a periodic structure may display finite widths because of:
 - a) Infinite sample size, i.e. infinite number of scattering planes?
 - b) Thermally excited phonons?
 - c) Only due to finite instrument resolution?
 - d) None of the above?
3. Which of the following type of radiation has the smallest penetration depth into materials:
 - a) X-rays?
 - b) Electrons?
 - c) Neutrons?
 - d) All radiation (a,b,c)) have the same penetration depth?
4. The intensity $I(q)$ of scattered x-rays from an isotropic sample is expressed as $I(q) \propto N S(q)$, where N is the number of particles in the scattering volume, $S(q)$ is the interference function, and $q = (4\pi \sin\theta)/\lambda$ is the magnitude of the scattering vector. In order to derive these expressions for $I(q)$ and q , we have assumed that:
 - a) The scattering is inelastic, i.e. there is a frequency shift in the scattering process from the primary to the scattered beam
 - b) The atoms are located at Bravais lattice positions
 - c) The atoms do not move during the scattering processes
 - d) None of the above
5. A self-avoiding random walk in 3 dimensions will create an open object with a fractal dimension D of:
 - a) $D = 3$
 - b) $D = 2$
 - c) $D < 1$
 - d) None of the above

6. Smectic order is the term used for:
- a) Positional order, but no orientational order
 - b) Isotropic order
 - c) Orientational order, but no positional order
 - d) None of the above
7. A pure elastic solid is described by:
- a) The Newtonian flow law: $\text{stress} = \text{viscosity} \times \text{shear-rate}$
 - b) Hooke's law: $\text{stress} = \text{modulus} \times \text{strain}$
 - c) Time-delayed relaxational behavior
 - d) An loss modulus that has a peak at the time characteristic for plastic flow
8. The term anelastic behavior refers to:
- a) Instantaneous response
 - b) Time-delayed response
 - c) Viscous response
 - d) None of the above
9. By orientational polarization, we mean the induced macroscopic polarization is a result of the sum of
- a) Permanent dipoles in the material that may reorient in an external electric field
 - b) Dipoles solely induced by an external electric field
 - c) Piezoelectric domains
 - d) None of the above
10. By distortional polarization, we mean the induced macroscopic polarization is a result of the sum of
- a) Permanent dipoles in the material that may reorient in an external electric field
 - b) Dipoles solely induced by an external electric field
 - c) Piezoelectric domains
 - d) None of the above

11. Piezoelectricity in a material means that:

- a) There is a quadratic relationship between elastic and electric fields in a material?
- b) There is a linear relationship between the elastic and electric fields in a material?
- c) There is no coupling between elastic and electric fields in a material?
- d) None of the above?

12. Diamagnetism:

- a) Diamagnetism is the term used for magnetism in materials with permanent magnetic dipoles
- b) The atomic diamagnetic susceptibility is independent of the number of electrons in the atom
- c) Diamagnetism is present in all materials
- d) None of the above?

13. Paramagnetism:

- a) Paramagnetism is present in all materials
- b) The paramagnetic susceptibility for a sample with localized magnetic moments is proportional to the temperature
- c) The paramagnetic susceptibility is in general small (compared to the diamagnetic susceptibility for example) and negative
- d) None of the above?

14. For second order phase transition, near the critical temperature T_C , critical fluctuations may become important for the observed behavior. Which of the following statements is true?

- a) Critical fluctuations are included in the classical Landau theory?
- b) Critical fluctuations are caused by infinitely large restoring forces preventing emergent dynamic and spontaneous regions of finite order parameter near T_C ?
- c) Critical fluctuations are considered to be objects with a size proportional to the lattice spacing of the material?
- d) None of the above?

15. Ferroelectricity is the results of

- a) Dipole-dipole interactions
- b) Distortional polarization only
- c) Piezoelectric couplings in the material
- d) None of the above

16. Ferromagnetism is basically a result of:

- a) Classical interaction between magnetic dipoles?
- b) Quantum mechanical exchange forces between spins of neighboring atoms?
- c) Quantum mechanical exchange of phonons resulting in paired electrons?
- d) None of the above?

17. Nuclear Magnetic Resonance (NMR) can be observed

- a) Only in ferromagnetic materials
- b) Only for spin 3/2 nuclei
- c) In all materials independent of nuclear spin states
- d) None of the above

18. Cooper pairs:

- a) Are bosons?
- b) Are formed by pairs of electrons bound together by exchanging photons?
- c) Are localized to within the unit cell?
- d) None of the above?

19. In a Dynamic Light Scattering (DLS) experiment, the dynamic structure factor $S(\mathbf{q}, t)$ is measured, where $\mathbf{q}$ is the scattering vector. Typical scattering curves from samples made from a dilute suspensions of Brownian diffusing colloidal spheres follow a simple relaxational behavior of the kind $S(\mathbf{q}, t) = \exp(-t/\tau(|\mathbf{q}|))$, where the relaxation time $\tau(|\mathbf{q}|) = C |\mathbf{q}|^2$. The proportionality constant C is

- a) $C = D_s$, the self diffusion constant?
- b) $C = 1/D_s$, the inverse of the self diffusion constant?
- c) C has no physical meaning, its just a fitting parameter to the data?
- d) $C = 1/\eta$, the inverse of the viscosity of the suspending liquid?

20. The widths of the Brillouin doublet peaks measured in Rayleigh - Brillouin experiments provide information about:

- a) Lifetimes of sound waves?
- b) Propagation velocities sound waves?
- c) Lifetimes of heat waves?
- d) Propagation velocities of heat waves?