



Contact during the exam:
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Exam in TFY4275 CLASSICAL TRANSPORT THEORY

Tuesday March 4th, 2008
14:15–16:00

Allowed help: Alternativ D
No written material allowed

This problem set consists of 2 pages.

Problem 1. Various topics

- a) Define the characteristic function (moment generating function), $G_\xi(k)$, of a random variable ξ defined by the probability distribution $p(\xi)$.
- b) How is the average $\langle e^\xi \rangle$ (ξ is a random variable) defined in terms of cumulants? Given the characteristic function, $G_\xi(k)$, what is then the cumulant generating function?
- c) Argue why the characteristic function of a *Gaussian* random variable, ξ_G , is given by

$$G_{\xi_G}(k) = \exp \left(i\mu k - \frac{\sigma^2 k^2}{2} \right),$$

where μ and σ denote the mean and standard deviation, respectively. (Note: No full mathematical derivation is asked for).

- d) What is the *main* difference between a random variable and a stochastic process? Give a real-life example of both of them.
- e) Given a general stochastic process, what is needed to fully define it?

Problem 2. Random Walk

We consider the sum of N random variables ξ_i

$$x_N = \sum_{i=1}^N \xi_i,$$

where *all* the ξ_i 's are *independent* and *identically* distributed (*i.e.* a random walk with steps ξ_i and position x_N at "time" $t = N$). The standard deviation of the (identical) steps (the random variables ξ_i) we denote by σ_ξ .

- a) Formulate with words the Central Limit Theorem (CLT).
- b) Use this to describe what the CLT predicts for the distribution of x_N , $p(x_N)$ when
 - i) $\sigma_\xi < \infty$?
 - ii) $\sigma_\xi = \infty$?No mathematical details needed (if you do not like to do so....).
- c) For the two cases mentioned above (i and ii), write down an expression for σ_{x_N} . When $\sigma_\xi < \infty$ express your answer in terms of σ_ξ and N .
- d) What is the characteristic function of the random variable x_N given the $G_\xi(k)$? What is the distribution of x_2 , denoted $p_2(x_2)$, expressed in terms of the distribution of ξ_i , $p(\xi)$.
- e) Now let us set $\xi_i = N(0, \sigma)$ for all i 's, *i.e.*, is Normally (Gaussian) distributed with zero mean and standard deviation $\sigma < \infty$. Show *mathematically* that your above statement for σ_{x_N} is correct.

Problem 3. Markov processes

- a) Define in terms of mathematics what a Markov process is, and explain with words what this definition means. What "order" of the joint probability distribution is *sufficient* to fully define a Markov process?
- b) What are the two *necessary* and *sufficient* conditions that have to be satisfied by *any* Markov process?
- c) Give three (different!) examples of Markov processes.

That is all for now guys. Good luck!