



NTNU – Trondheim
Norwegian University of
Science and Technology

Examination paper for TFY4280 Signal Processing

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Examination date: 02.06.2017

Examination time (from-to): 0900 - 1300

Permitted examination support material:

- Simple calculator (according to NTNU exam regulations)
- K. Rottmann: Matematisk formelsamling or an equivalent, for example: Formulaires et tables: mathématiques, physique, chimie. Editions du Tricorne, Genève.
- Barnett and Cronin: Mathematical formulae
- Carl Angell og Bjørn Ebbe Lian: Fysiske størrelser og enheter, navn og symboler

Other information about the exam paper:

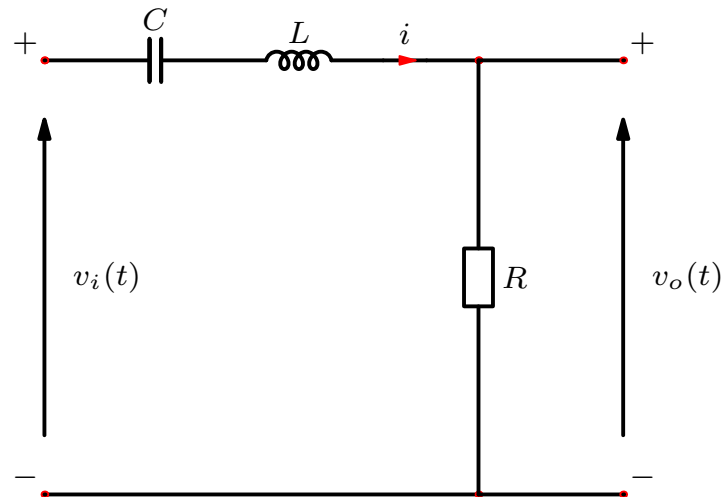
- Language: English
- Number of pages (including this page and attachments): 5
- Answer must be written in English or Norwegian. Number of points given to each question is given in bold font. The maximum score for the exam is **100p**.
- **Attachment:** 2 pages with transform tables and properties.

Checked by:

Date:

Signature:

Q1. (35p) Consider a simple circuit shown below, for which $R = 10$, $C = 0.0235$ and $L = 10$ in the appropriate s.i. units.

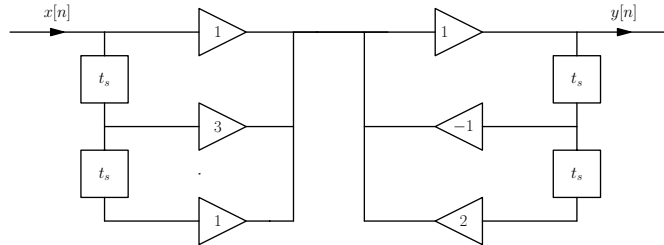


- A.** Write a differential equation which relates the output $v_o(t)$ with the input $v_i(t)$ for this system. **(5p)**
- B.** Give a definitions of an impulse response function, a transfer function and a frequency response function. **(5p)**
- C.** Calculate the impulse response function for the system given above. It is sufficient to express $h(t)$ as a sum of complex exponential functions. Explain how you would used this function to calculate the output signal $y(t)$ for a given input signal $x(t)$. **(10p)**
- D.** Calculate the transfer function and the frequency response function of this system. For the frequency response function, you do not need to arrive at a elegant expressions. **(5p)**
- E.** Draw a zero/poles diagram. **(5p)**
- F.** What kind of circuit is this? **(5p)**

Q2. (10p) Find the z-transform of the following signal:

$$x[n] = \{0, 1, 2, 3, \underline{4}, 3, 2, 1, 0\}$$

Q3. (20p) Consider a discrete-time system described by a block diagram shown below.



- A.** Use this block diagram to calculate a discrete-time impulse response function for this system. Calculate for $n < 6$. **(10p)**
- B.** Use a method of choice to determine output signal $y[n]$ for $n < 8$ and the input signal given below. **(10p)**

$$x[n] = \{1, 0, 1, 0\} \quad \text{and } 0 \text{ otherwise.}$$

Q4. (10p) Determine the convolution

$$y(t) = e^{-at}\epsilon(t) * \epsilon(t)$$

using direct method (definition) and using Fourier transform method.

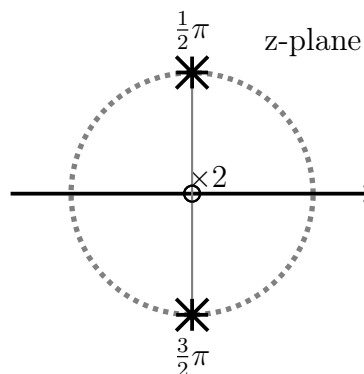
Q5. (25p)

- A.** Use definition of of a unilateral z-transform ($\mathcal{Z}$) to determine

$$\mathcal{Z} \{u[n]e^{\beta n}\} =$$

and then use this to calculate $\mathcal{Z} \{u[n]\cos(bn)\}$ **(10p)**

- B.** Zeros/poles diagram on the z-plane for a transfer function $H(z)$ of a LTI discrete-time system is given below. Use this diagram to find the difference equation which describes this system. **(15p)**



Appendix B.1 Bilateral Laplace Transform Pairs

$x(t)$	$X(s) = \mathcal{L}\{x(t)\}$	ROC
$\delta(t)$	1	$s \in \mathbb{C}$
$\varepsilon(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
$e^{-at}\varepsilon(t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} > \text{Re}\{-a\}$
$-e^{-at}\varepsilon(-t)$	$\frac{1}{s+a}$	$\text{Re}\{s\} < \text{Re}\{-a\}$
$t\varepsilon(t)$	$\frac{1}{s^2}$	$\text{Re}\{s\} > 0$
$t^n\varepsilon(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}\{s\} > 0$
$t e^{-at}\varepsilon(t)$	$\frac{1}{(s+a)^2}$	$\text{Re}\{s\} > \text{Re}\{-a\}$
$t^n e^{-at}\varepsilon(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\text{Re}\{s\} > \text{Re}\{-a\}$
$\sin(\omega_0 t)\varepsilon(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
$\cos(\omega_0 t)\varepsilon(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
$e^{-at}\cos(\omega_0 t)\varepsilon(t)$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > \text{Re}\{-a\}$
$e^{-at}\sin(\omega_0 t)\varepsilon(t)$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$	$\text{Re}\{s\} > \text{Re}\{-a\}$
$t\cos(\omega_0 t)\varepsilon(t)$	$\frac{s^2 - \omega_0^2}{(s^2 + \omega_0^2)^2}$	$\text{Re}\{s\} > 0$
$t\sin(\omega_0 t)\varepsilon(t)$	$\frac{2\omega_0 s}{(s^2 + \omega_0^2)^2}$	$\text{Re}\{s\} > 0$

Appendix B.3 Fourier Transform Pairs

$x(t)$	$X(j\omega) = \mathcal{F}\{x(t)\}$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\delta(t)$	$j\omega$
$\frac{1}{T} \text{III}\left(\frac{t}{T}\right)$	$\text{III}\left(\frac{\omega T}{2\pi}\right)$
$\varepsilon(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\text{rect}(at)$	$\frac{1}{ a } \text{si}\left(\frac{\omega}{2a}\right)$
$\text{si}(at)$	$\frac{\pi}{ a } \text{rect}\left(\frac{\omega}{2a}\right)$
$\frac{1}{t}$	$-j\pi \text{sign}(\omega)$
$\text{sign}(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos(\omega_0 t)$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\sin(\omega_0 t)$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$e^{-\alpha t }, \alpha > 0$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
$e^{-a^2 t^2}$	$\frac{\sqrt{\pi}}{a} e^{-\frac{\omega^2}{4a^2}}$

Appendix B.2 Properties of the Bilateral Laplace Transform

$x(t)$	$X(s) = \mathcal{L}\{x(t)\}$	ROC
Linearity $Ax_1(t) + Bx_2(t)$	$AX_1(s) + BX_2(s)$	$\text{ROC} \supseteq \text{ROC}\{X_1\} \cap \text{ROC}\{X_2\}$
Delay $x(t - \tau)$	$e^{-s\tau}X(s)$	not affected
Modulation $e^{at}x(t)$	$X(s - a)$	$\text{Re}\{a\}$ shifted by $\text{Re}\{a\}$ to the right
'Multiplication by t ', Differentiation in the frequency domain $tx(t)$	$-\frac{d}{ds}X(s)$	not affected
Differentiation in the time domain $\frac{d}{dt}x(t)$	$sX(s)$	$\text{ROC} \supseteq \text{ROC}\{X\}$
Integration $\int_{-\infty}^t x(\tau)d\tau$	$\frac{1}{s}X(s)$	$\text{ROC} \supseteq \text{ROC}\{X\} \cap \{s : \text{Re}\{s\} > 0\}$
Scaling $x(at)$	$\frac{1}{ a }X\left(\frac{s}{a}\right)$	ROC scaled by a factor of a

Appendix B.4 Properties of the Fourier Transform

	$x(t)$	$X(j\omega) = \mathcal{F}\{x(t)\}$
Linearity	$Ax_1(t) + Bx_2(t)$	$AX_1(j\omega) + BX_2(j\omega)$
Delay	$x(t - \tau)$	$e^{-j\omega\tau}X(j\omega)$
Modulation	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
'Multiplication by t ', Differentiation in the frequency domain $tx(t)$	$tx(t)$	$-\frac{dX(j\omega)}{d(j\omega)}$
Differentiation in the time domain $\frac{dx(t)}{dt}$	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^t x(\tau)d\tau$	$X(j\omega) \left[\pi\delta(\omega) + \frac{1}{j\omega} \right]$ $= \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Scaling	$x(at)$	$\frac{1}{ a }X\left(\frac{j\omega}{a}\right), \quad a \in \mathbb{R} \setminus \{0\}$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega) \cdot X_2(j\omega)$
Multiplication	$x_1(t) \cdot x_2(t)$	$\frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$
Duality	$x_1(t)$ $x_2(jt)$	$x_2(j\omega)$ $2\pi x_1(-\omega)$
Symmetry relations	$x(-t)$ $x^*(t)$ $x^*(-t)$	$X(-j\omega)$ $X^*(-j\omega)$ $X^*(j\omega)$
Parseval theorem	$\int_{-\infty}^{\infty} x(t) ^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$

Appendix B.6 Properties of the z -Transform

Property	$x[k]$	$X(z)$	ROC
Linearity	$ax_1[k] + bx_2[k]$	$aX_1(z) + bX_2(z)$	$\text{ROC} \supseteq \text{ROC}\{X_1\} \cap \text{ROC}\{X_2\}$
Delay	$x[k - \kappa]$	$z^{-\kappa}X(z)$	$\text{ROC}\{x\}$; separate consideration of $z = 0$ and $z \rightarrow \infty$
Modulation	$a^k x[k]$	$X\left(\frac{z}{a}\right)$	$\text{ROC} = \left\{z \mid \frac{z}{a} \in \text{ROC}\{x\}\right\}$
Multiplication by k	$kx[k]$	$-z \frac{dX(z)}{dz}$	$\text{ROC}\{x\}$; separate consideration of $z = 0$
Time inversion	$x[-k]$	$X(z^{-1})$	$\text{ROC} = \{z \mid z^{-1} \in \text{ROC}\{x\}\}$
Convolution	$x_1[k] * x_2[k]$	$X_1(z) \cdot X_2(z)$	$\text{ROC} \supseteq \text{ROC}\{x_1\} \cap \text{ROC}\{x_2\}$
Multiplication	$x_1[k] \cdot x_2[k]$	$\frac{1}{2\pi j} \oint X_1(\zeta) X_2\left(\frac{z}{\zeta}\right) \frac{1}{\zeta} d\zeta$	multiply the limits of the ROC

Appendix B.5 Two-sided z -Transform Pairs

$x[k]$	$X(z) = \mathcal{Z}\{x[k]\}$	ROC
$\delta[k]$	1	$z \in \mathbf{C}$
$\varepsilon[k]$	$\frac{z}{z-1}$	$ z > 1$
$a^k \varepsilon[k]$	$\frac{z}{z-a}$	$ z > a $
$-a^k \varepsilon[-k-1]$	$\frac{z}{z-a}$	$ z < a $
$k\varepsilon[k]$	$\frac{z}{(z-1)^2}$	$ z > 1$
$ka^k \varepsilon[k]$	$\frac{az}{(z-a)^2}$	$ z > a $
$\sin(\Omega_0 k) \varepsilon[k]$	$\frac{z \sin \Omega_0}{z^2 - 2z \cos \Omega_0 + 1}$	$ z > 1$
$\cos(\Omega_0 k) \varepsilon[k]$	$\frac{z(z - \cos \Omega_0)}{z^2 - 2z \cos \Omega_0 + 1}$	$ z > 1$