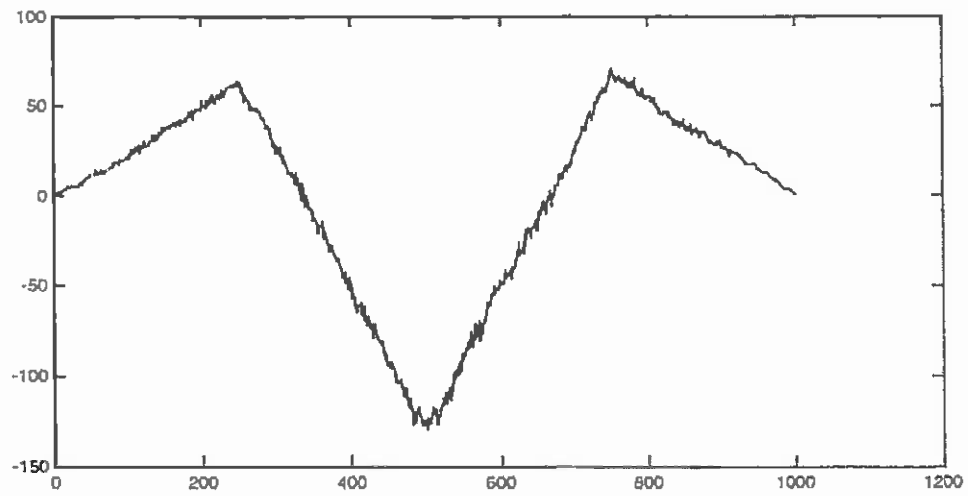
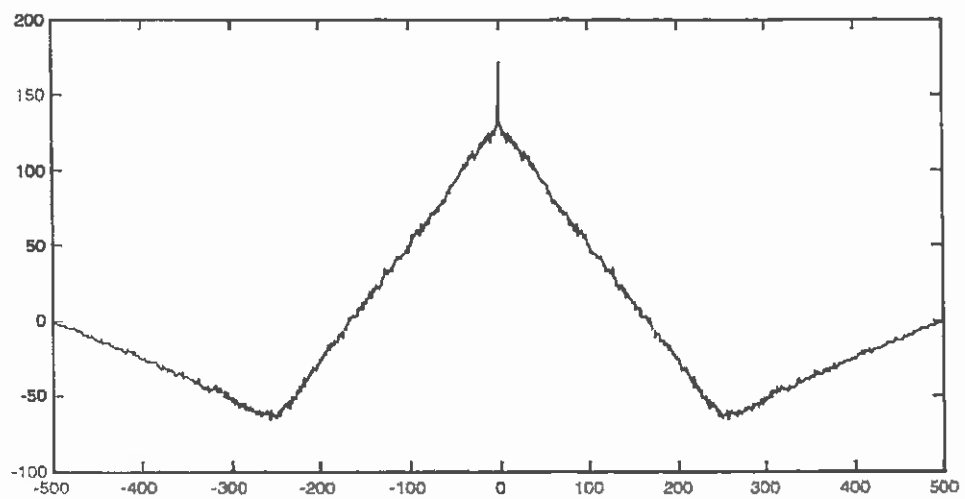


Suggested solutions Examination May 2010



Solution A1



Solution A2

$$A3. \quad H(s) = \frac{2}{(s+2)^2}$$

Unit step excitation $X(s) = \frac{1}{s}$

$$\Rightarrow Y(s) = \frac{2}{s(s+2)^2}$$

Partial fraction expansion:

$$\frac{A}{s} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)}$$

$$A = \frac{2}{(0+2)^2} = \frac{1}{2} \quad ; \quad B = \left(\frac{2}{(-2)^2} \right) = -1$$

$$\text{Thus,} \quad \frac{0.5}{s} - \frac{1}{(s+2)^2} + \frac{C}{(s+2)} = \frac{2}{s(s+2)^2}$$

$$0.5(s+2)^2 - s + C s(s+2) = 2$$

$$0.5(s^2 + 4s + 4) - s + C s^2 + 2Cs = 2$$

$$s^2: 0.5 + C = 0 \quad \Rightarrow \quad C = -0.5$$

$$s^1: 2s - s + 2C \cdot s = 0 \quad \text{OK}$$

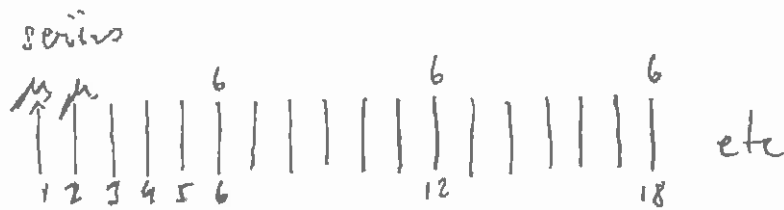
$$s^0: 2 = 2 \quad \text{OK}$$

$$\therefore Y(s) = \frac{0.5}{s} - \frac{1}{(s+2)^2} - \frac{0.5}{(s+2)}$$

from table:

$$\underline{y(t) = \left(0.5 - 0.5 \cdot e^{-2t} - t e^{-2t} \right) \cdot u(t)}$$

A4.



for a series, the expected random number μ appears @ throw $1 \rightarrow 5$

$$\mu = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

$$\Rightarrow \text{for series } 1 \rightarrow 6 \quad \frac{6 + 5 \cdot 3.5}{6} = \frac{47}{12} = \overline{\mu_y[k]} (\sim 3.92)$$

and similar for all series in blocks of 6

Similarly we get

$$E\{y^2[k]\}_{\text{normal}} = \frac{1+4+9+16+25+36}{6} = \frac{91}{6}$$

so for the loaded die we get

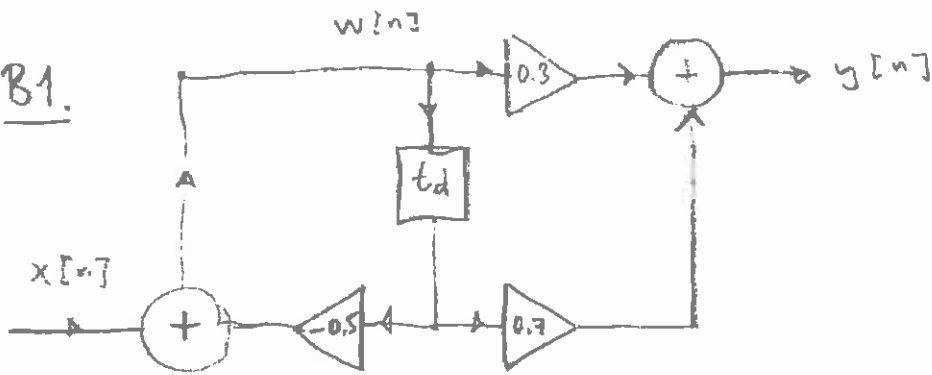
$$\frac{1}{6} \left(5 \cdot \frac{91}{6} + 6^2 \right) = \frac{671}{36} (\sim 18.64)$$

$$\text{So that for loaded die } \Delta_x^2 = \frac{671}{36} - \left(\frac{47}{12} \right)^2 = \frac{475}{144}$$

$$\text{compare with 'normal' die } \Delta^2 = \frac{91}{6} - 3.5^2 = \frac{25}{12} (\sim 2.08) \quad (\sim 3.30)$$

$$\underline{\text{Ans.}} \quad \overline{\mu_y[k]} = \frac{47}{12} \quad ; \quad \Delta_y[k] = \frac{5\sqrt{19}}{12} \sim 1.81$$

B1.



a)

$$w[n] = x[n] - 0.5 w[n-1]$$

$\xrightarrow{z}$

$$W = X - 0.5 z^{-1} W$$

$$y[n] = 0.3 w[n] + 0.7 w[n-1]$$

$$Y = 0.3 W + 0.7 z^{-1} W$$

$$\Rightarrow W(1 + 0.5 z^{-1}) = X \Rightarrow W = X \cdot \frac{1}{(1 + 0.5 z^{-1})}$$

$$Y = (0.3 + 0.7 z^{-1}) W$$

$$\Rightarrow Y = \frac{(0.3 + 0.7 z^{-1})}{(1 + 0.5 z^{-1})} X$$

$$Y(1 + 0.5 z^{-1}) = 0.3 X + 0.7 z^{-1} X \quad \xrightarrow{z^{-1}}$$

$$\Rightarrow \underline{y[n] = 0.3 x[n] + 0.7 x[n-1] - 0.5 y[n-1]}$$

$$b) H(z) = \frac{Y(z)}{X(z)} = \frac{0.3 + 0.7 z^{-1}}{1 + 0.5 z^{-1}} = \frac{0.3z + 0.7}{z + 0.5}$$

B1. cont

c) unit step response

$$Y(z) = H(z) \cdot \frac{z}{(z-1)} = \frac{(0.3z + 0.7)z}{(z+0.5)(z-1)}$$

long division gives

$$\begin{array}{r} 0.3 + 0.85z^{-1} + 0.575z^{-2} + 0.7125z^{-3} + 0.64375z^{-4} \\ z^2 - 0.5z - 0.5 \overline{) 0.3z^2 + 0.7z} \\ \underline{0.3z^2 - 0.15z - 0.15} \\ 0 0.85z + 0.15 \\ \underline{0.85z - 0.425 - 0.425z^{-1}} \\ 0 0.575 + 0.425z^{-1} \\ \underline{0.575 - 0.2875z^{-1} - 0.2875z^{-2}} \\ 0 0.7125z^{-1} + 0.2875z^{-2} \\ \underline{0.7125z^{-1} - 0.35625z^{-2}} \\ 0 -0.35625z^{-2} \end{array}$$

$$0.64375z^{-2} + 0.35625z^{-3}$$

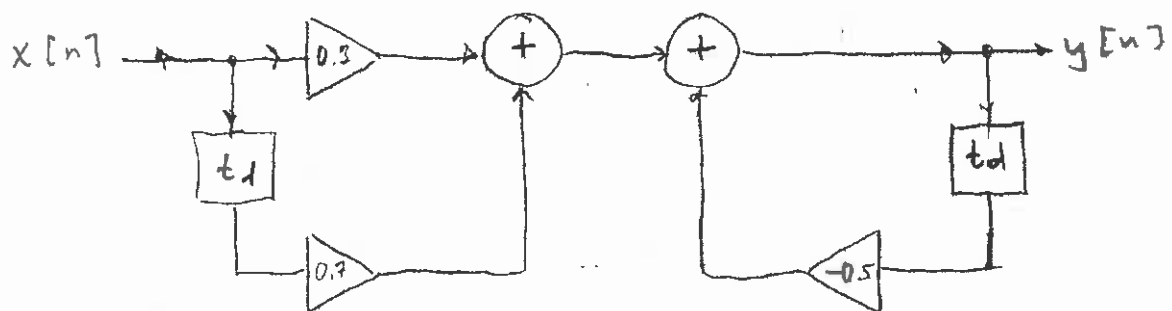
$$0.64375z^{-2} - 0.321875z^{-3} - 0.321875z^{-4}$$

$$0 0.678125z^{-3} + 0.321875z^{-4}$$

$\Rightarrow$ Sequence: $\{0.3, 0.85, 0.575, 0.7125, \dots\}$

d) the difference equation in (a) gives

$$y[n] = 0.3x[n] + 0.7x[n-1] - 0.5y[n-1]$$



B2

a) Prove the time-shift property of the Fourier transform

$$f(t-t_0) \rightarrow F(\omega) e^{-i\omega t_0}$$

From the definition $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$

now let $t \rightarrow t-t_0$

$$F(\omega) = \int_{-\infty}^{\infty} f(t-t_0) e^{-i\omega t} dt$$

$$\text{let } t-t_0 = t' \Rightarrow dt' = dt \quad ; \quad \text{limits } \rightarrow \pm \infty$$

$$\Rightarrow F(\omega) = \int_{-\infty}^{\infty} f(t') e^{-i\omega(t'+t_0)} dt' = e^{-i\omega t_0} \int_{-\infty}^{\infty} f(t') e^{-i\omega t'} dt'$$

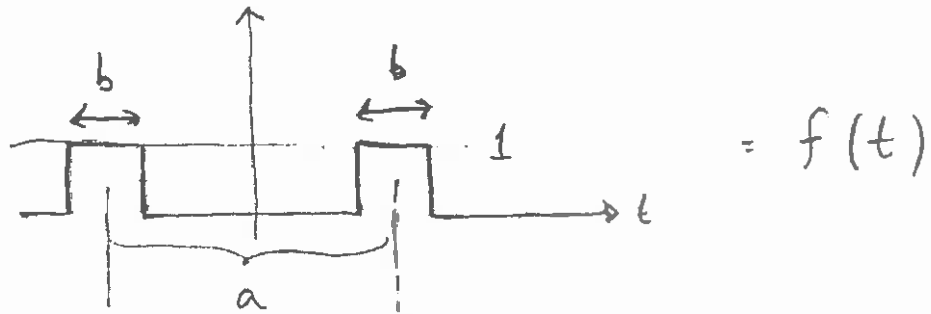
$$\therefore e^{-i\omega t_0} \cdot F(\omega)$$

Q.E.D

$$F(\omega)$$

B2.

b)



The FT is linear, thus we can add the contribution for each time-slot and make use of the shift-theorem as deduced in (a)

$\text{rect}(t/b) \xrightarrow{\text{FT}} b \cdot \text{sinc}\left(\frac{\omega b}{2}\right)$

shift-theorem gives that

$$F\{f(t)\} = b \cdot \text{sinc}\left(\frac{\omega b}{2}\right) \cdot \left\{ e^{i\frac{\omega a}{2}} + e^{-i\frac{\omega a}{2}} \right\}$$
$$= \underline{\underline{2b \cos\left(\frac{\omega a}{2}\right) \text{sinc}\left(\frac{\omega b}{2}\right)}}$$

B2 cont...
check by brute force....

$$\int_{-\frac{a}{2} - \frac{b}{2}}^{-\frac{a}{2} + \frac{b}{2}} e^{-i\omega t} dt + \int_{\frac{a}{2} - \frac{b}{2}}^{\frac{a}{2} + \frac{b}{2}} e^{-i\omega t} dt = F(\omega) =$$

$$= \frac{1}{(-i\omega)} \left[e^{-i\omega t} \right]_{-\frac{a}{2} - \frac{b}{2}}^{-\frac{a}{2} + \frac{b}{2}} + \frac{1}{(-i\omega)} \left[e^{-i\omega t} \right]_{\frac{a}{2} - \frac{b}{2}}^{\frac{a}{2} + \frac{b}{2}} =$$

$$= \frac{(-2)}{2i\omega} \left[e^{\frac{i\omega a}{2}} \cdot e^{-\frac{i\omega b}{2}} - e^{\frac{i\omega a}{2}} \cdot e^{\frac{i\omega b}{2}} + e^{-\frac{i\omega a}{2}} \cdot e^{-\frac{i\omega b}{2}} - e^{-\frac{i\omega a}{2}} \cdot e^{\frac{i\omega b}{2}} \right]$$

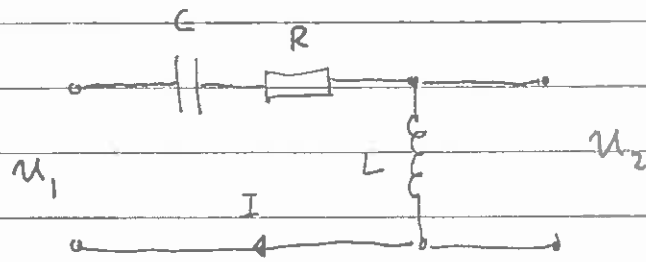
$$= \frac{-2}{2i\omega} \left[e^{\frac{i\omega a}{2}} \left(e^{-\frac{i\omega b}{2}} - e^{\frac{i\omega b}{2}} \right) + e^{-\frac{i\omega a}{2}} \left(e^{-\frac{i\omega b}{2}} - e^{\frac{i\omega b}{2}} \right) \right]$$

$$= \left(\frac{e^{\frac{i\omega b}{2}} - e^{-\frac{i\omega b}{2}}}{2i} \right) \cdot \frac{2}{\omega} \left(e^{\frac{i\omega a}{2}} + e^{-\frac{i\omega a}{2}} \right)$$

$$= \frac{2 \cdot 2^{\frac{b}{2}}}{(\omega^{\frac{b}{2}})} \cdot \cos\left(\frac{\omega a}{2}\right) \cdot \sin\left(\frac{\omega b}{2}\right) = 2b \cdot \cos\left(\frac{\omega a}{2}\right) \operatorname{sinc}\left(\frac{\omega b}{2}\right)$$

Q.E.D

B9:



a) impedance method gives $I = \frac{U_1}{\frac{1}{sC} + R + sL}$

$$U_2 = I \cdot sL$$

$$\Rightarrow U_2 = U_1 \cdot \frac{sL}{\frac{1}{sC} + R + sL}$$

$$\Rightarrow \frac{U_2}{U_1} = H(s) = \frac{s^2 LC}{s^2 LC + sRC + 1} = \frac{s^2}{\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)}$$

b) $H(s) = \frac{s^2}{(s+a)^2}$?

$$s^2 + s\frac{R}{L} + \frac{1}{LC} = \left(s + \frac{R}{2L}\right)^2 + \frac{1}{LC} - \frac{R^2}{4L^2}$$

holds if $\frac{1}{LC} = \frac{R^2}{4L^2} \Rightarrow \underline{\underline{C = \frac{4L}{R^2}}}$

$$a = \frac{R}{2L}$$

B9:

$$c) \quad H(\omega) = \frac{(i\omega)^2}{(i\omega + a)^2}$$

$$|H(\omega)| = \frac{\omega^2}{(\omega^2 + a^2)} = \frac{1}{2} \text{ @ cut off}$$

$$\text{cut-off frequency for } \omega = 100 \quad \left(20 \cdot \log H(100) = -3 \right)$$

$$\Rightarrow \omega = 100 \Rightarrow \frac{R}{2L} = 12,86$$

$$a^2 = (\sqrt{2}-1)100 \Rightarrow 6,43 = \frac{R^2}{2L}$$

d) plot $|H(\omega)|$ log scale 0.1; 1; 10; 100; 1000

$$0.1 \quad 20 \cdot \log \frac{0.1}{0.1+100} \sim 20 \cdot (-3) = -60$$

$$1.0 \quad 20 \cdot \log \frac{1}{1+100} \sim 20 \cdot (-2) = -40$$

$$10 \quad 20 \cdot \log \frac{10}{10+100} \sim 20 \cdot (-1) = -20$$

$$100 \quad 20 \cdot \log \frac{100}{100+100} \sim -3$$

$$1000 \quad 20 \cdot \log \frac{1000}{1000+100} \sim 20 \cdot 0 = 0$$

