Some useful expressions and values.

$$
\begin{aligned}
I &= NE \\
I &= I_0 e^{-\int \mu(x)dx} \\
S &= \varepsilon A(I_P + I_S) \\
C &= \frac{I_1 - I_2}{I_1} \\
k &= \frac{S_1 - S_2}{STD_s} \\
D &= EN_0(\frac{\mu_{en}}{\rho}) \\
\varepsilon &= \frac{N_{out}}{N_{in}} \\
p(s,\theta) &= -ln\frac{I}{I_0} = \int_A^B \mu(s,t)dt \\
HU &= 1000 \cdot \frac{\mu - \mu_w}{\mu_w} \\
R_c &= \frac{d(L+z)}{L} \\
g &= \frac{d^2}{4\pi L^2}\frac{d^2}{(d+t)^2} \\
t &= \frac{6d}{\mu L - 3} \\
S &= S_0(1 - e^{-TR/T1})e^{-TE/T2} \\
S &= \frac{S_0(1 - e^{-TR/T1}) sin\theta e^{-TE/T2}}{1 - cos\theta e^{-TR/T1}} \\
f\lambda &= c \\
c &= c_0 + v\beta \\
\beta &= 1 + B/2A \\
f_d &= \frac{\pm 2 f_0 v cos\theta}{c}
\end{aligned}
$$

Problem 1: X-ray and CT Imaging

1a

The following three detector materials are available:

1. Unstructured CsI.
2. Structured CsI.
3. Amorphous Se.

Mass attenuation coefficents are found in figure 1. Density is around 4.5 g/cm^3 for both materials.

- Which detector material would you choose for use in skeletal X-ray at an effective energy of 50 keV? Assume a fixed detector thickness should be used. Justify your answer.
- Estimate the detector thickness required to achieve 99% detector efficiency for your chosen detector material?
- Calculate the image SNR if the incoming X-ray intensity is 8000 MeV/cm^2 and the pixel size is 0.2x0.2 mm^2.

1b

Suggest a computer program (in pseudo-code) for a filtered back-projection reconstruction algorithm for two different cases:

- 1D filtering in the image domain.
- 2D filtering in the Fourier domain.

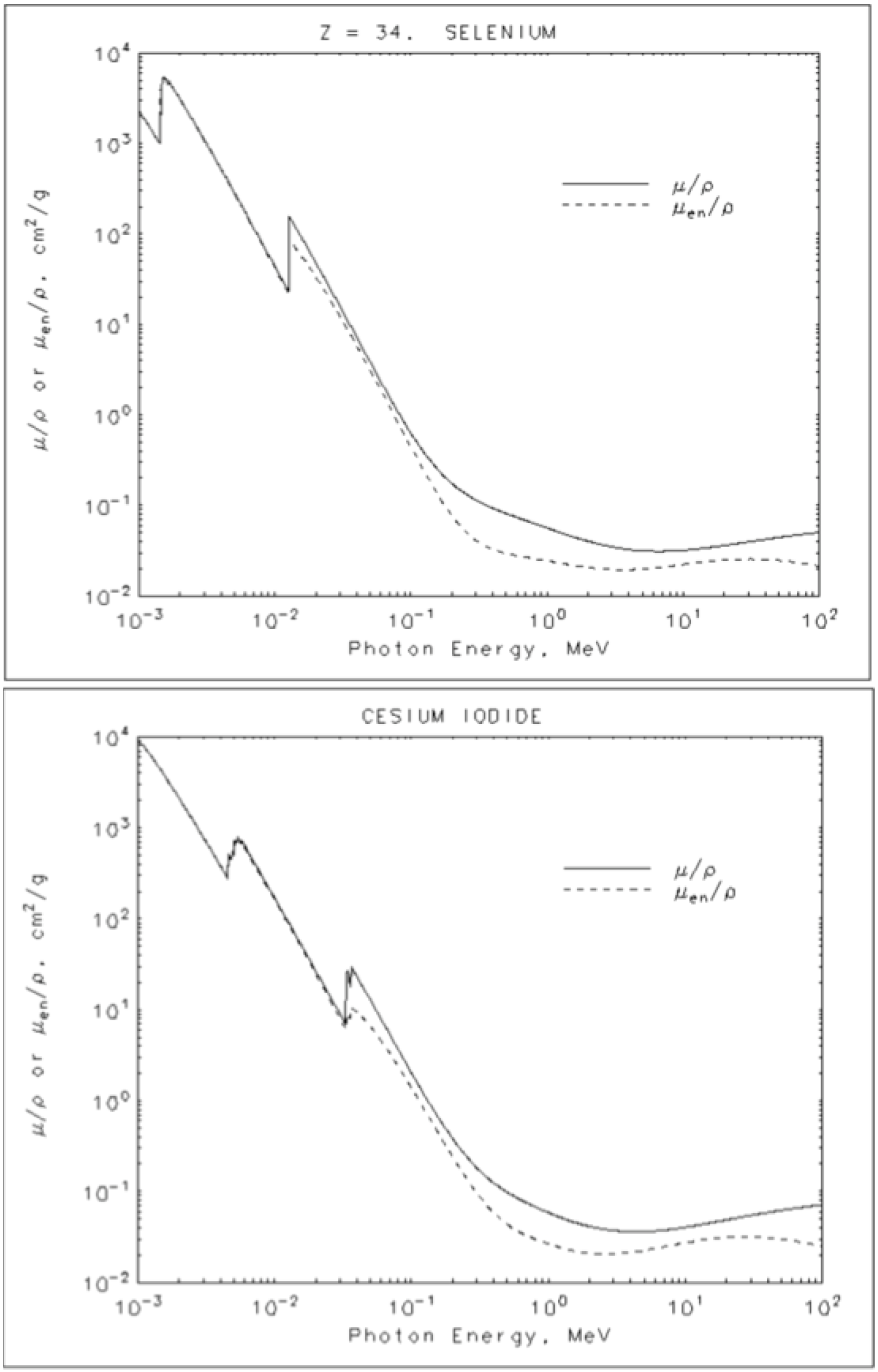

Figure 1: Mass attenuation coefficients, problem 1a.

Problem 2: Nuclear Medicine Imaging

2a

Draw and explain the following three energy spectra in SPECT imaging, assuming initial emission of mono-energetic gamma-photons at energy E_γ inside a human body:

- Energy spectrum of the radiation emerging from the body.
- Spectrum of absorbed energy in the scintillator.
- Final spectrum as measured by the gamma camera.

2b

Given the following two parallel hole collimators:

1. Hole length = 36 mm, hole diameter = 3.4 mm, septal thickness = 0.5 mm.
2. Hole length = 40 mm, hole diameter = 1.8 mm, septal thickness = 0.3 mm.

- Which collimator would you choose for single-photon emission imaging in an application where high spatial resolution is the highest priority. Justify your answer with calculations.
- What is the disadvantage of your chosen collimator?

2c

- Describe the typical detector design for PET, including the scintillator crystals and photo detector geometry.
- In particular, explain how localization is achieved.

Problem 3: Magnetic Resonance Imaging

3a

- Describe the difference between R2 and R2' in terms of their physical origin.
- Suggest a measurement procedure in order to estimate R2'.

3b

Derive the signal equation for the FID sequence in the case of arbitrary flip angle $< 90°$. Make sure to explain the concept of steady-state as part of the derivation.

3c

- Describe the effect of a applying linear magnetic field gradient during signal acquisition.
- Express the relation between the gradient G and wavenumber k.

Problem 4: Ultrasound Imaging

4a

Starting from the following relation:

$$p = -\kappa \frac{\partial u}{\partial x}$$

where p is the pressure and u is the longitudinal displacement, derive the 1D acoustic wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

Define the wave velocity c.

4b

Explain the physical origin of speckle in pulse-echo imaging.