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Contact during the exam:
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EXAM
TFY4340 MESOSCOPIC PHYSICS
Wednesday May 12 2010, 0900 - 1300
English

Remedies: C

- K. Rottmann: Mathematical formulae
- Approved calculator with empty memory (Citizen SR-270X, HP30S, or similar).

Pages 2 – 6: Questions 1 – 3. The three questions are relatively unrelated and may be answered in any order. Also, many of the subquestions within a given question may be answered independently from the others.

Notation: Vectors are given in **bold italic**. Unit vectors are given with a hat above the symbol.

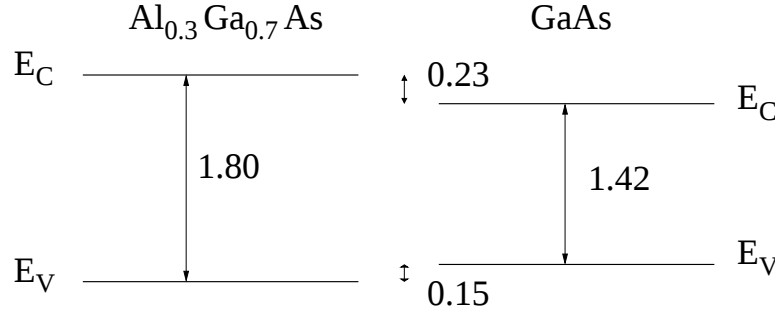
Some constants:

Electron mass: $m = 9.1 \cdot 10^{-31}$ kg. Elementary charge: $e = 1.6 \cdot 10^{-19}$ C.

Boltzmann constant: $k_B = 1.38 \cdot 10^{-23}$ J/K. Planck constant: $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js.

The grades will be available no later than May 28.

QUESTION 1



The figure above shows the energy band characteristics at the Γ -point ($\mathbf{k} = 0$) in the two semiconductors $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and GaAs , with bandgaps and conduction and valence band offsets in the unit eV.

An interface is formed between (uniformly) n -doped $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and undoped ("intrinsic") GaAs . We assume that the Fermi level (or: chemical potential) μ in bulk n - $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ lies 0.13 eV below the conduction band edge E_C .

- Make a sketch of the resulting energy band diagram near the interface when equilibrium has been established. Explain briefly the difference between your figure and the figure given above.
- Concerning the properties of the two-dimensional electron gas (2DEG) that is now formed in GaAs next to the interface, what would be the benefit of introducing an undoped layer of $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ between n - $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and GaAs ?

Let $z = 0$ denote the location of the interface. To a first approximation, the conduction band edge may, in the vicinity of the interface, be described by the potential $V(z) = Fz$ for $z > 0$ and $V(z) = \infty$ for $z < 0$. Here, F is a constant. In the xy -plane, the electrons move essentially as free particles, with kinetic energy $\hbar^2 k^2 / 2m^*$, and effective mass $m^* = m^*(\text{GaAs}) = 0.067m$ due to the periodic potential felt by the electrons. The total energy of the two-dimensional (2D) subbands is therefore

$$E_n(\mathbf{k}) = E_n + \frac{\hbar^2 k^2}{2m^*}, \quad n = 1, 2, 3, \dots,$$

with

$$\mathbf{k} = k_x \hat{x} + k_y \hat{y}.$$

Here, zero energy is chosen at the conduction band edge in GaAs when $z \rightarrow 0$, i.e., at the bottom of the triangular potential well. The corresponding wave functions are

$$\Psi_{n\mathbf{k}}(\mathbf{r}) = \Phi_n(z) u_{\mathbf{k}}(x, y) e^{i(k_x x + k_y y)},$$

with the (x, y) -dependent part on Bloch form. The Schrödinger equation is now separable.

c) Show that the resulting equation for Φ_n may be written in dimensionless form,

$$\frac{d^2 \Phi_n}{d\xi^2} - \xi \Phi_n = -\tilde{E} \Phi_n,$$

with $\xi = \kappa z$, $\tilde{E} = E\kappa/F$, and $\kappa = (2m^*F/\hbar^2)^{1/3}$.

d) The (dimensionless) eigenvalues of this equation are approximately $\tilde{E}_1 = 2.34$, $\tilde{E}_2 = 4.09$, $\tilde{E}_3 = 5.52$, Show that with $F = 10$ meV/nm, and a Fermi level $\mu = 0.10$ eV, only the lowest 2D subband, $E_1(\mathbf{k})$, will be occupied by electrons.

The density of states (DOS), i.e., the number of available quantum states pr unit energy, $D_2(E)$, is independent of the energy E in a 2DEG. Let us prove this statement.

e) Assume that your sample is of size $L \times L$ in the xy -plane. Then, taking into account the spin degeneracy of two and the absence of valley degeneracy (i.e., $g_S = 2$, $g_V = 1$) near the Γ -point, argue that the density of states in (the 2D) $\mathbf{k}$ -space is

$$D_2(\mathbf{k}) = \frac{L^2}{2\pi^2}.$$

f) Further, argue that the number of states $N_2(k)$ with wave number less than k (in absolute value) is then

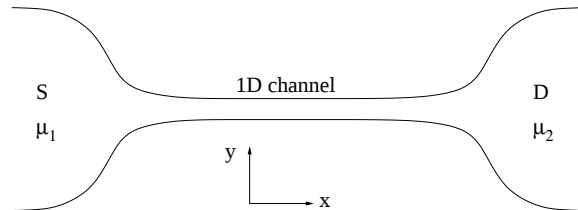
$$N_2(k) = \frac{(kL)^2}{2\pi}.$$

g) Use this result to write down $N_2(E)$ in "energy space", assuming $E = 0$ at the bottom of the 2D subband. Finally, show that

$$D_2(E) = \frac{m^* L^2}{\pi \hbar^2},$$

a constant, as advertised.

Next, we will consider electron transport through a 1D channel connecting a 2D source (S) at chemical potential (Fermi level) μ_1 and a 2D drain (D) at chemical potential μ_2 ($\mu_2 < \mu_1$):



The electric current from S to D, due to transverse subband j in the 1D channel, is

$$I_j^+ = (-e) \int_{E_j^t}^{\mu_1} dE \rho_j^+(E) v_j(E) T_j(E).$$

Here, $\rho_j^+(E)$ is the 1D DOS pr unit length for states with positive group velocity $v_j(E)$ along the 1D channel, and $T_j(E)$ is the probability of being transmitted through the 1D channel in subband j with energy E . We assume low temperatures.

h) Verify that this expression for I_j^+ has the correct unit (A).

i) From this expression for I_j^+ , and an analogous expression for the current from D to S, I_j^- , derive the Landauer formula for the total conductance $G = I/V$ of the 1D channel,

$$G = \frac{2e^2}{h} \sum_j T_j(E_F).$$

Here, I is the total net current when a voltage V is applied between S and D, and we are assuming linear response, i.e., $\mu_1 \simeq \mu_2 \simeq E_F$. The 1D DOS is $D_1(E) = \sqrt{2m^*}L/\pi\hbar\sqrt{E}$ for a system of length L .

The figure below is copied from the paper *Quantized Conductance of Point Contacts in a Two-Dimensional Electron Gas*, by B. J. van Wees et al (*Phys Rev Lett* **60**, 848 (1988)):

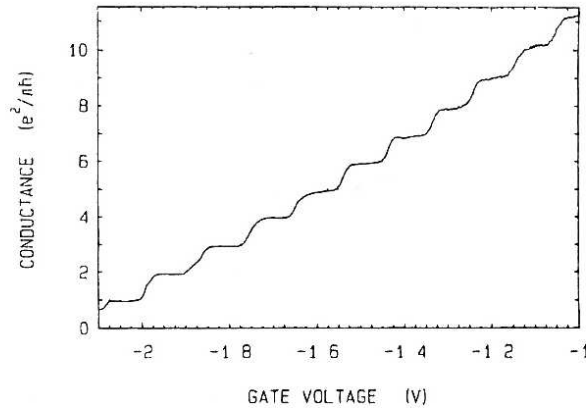


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.

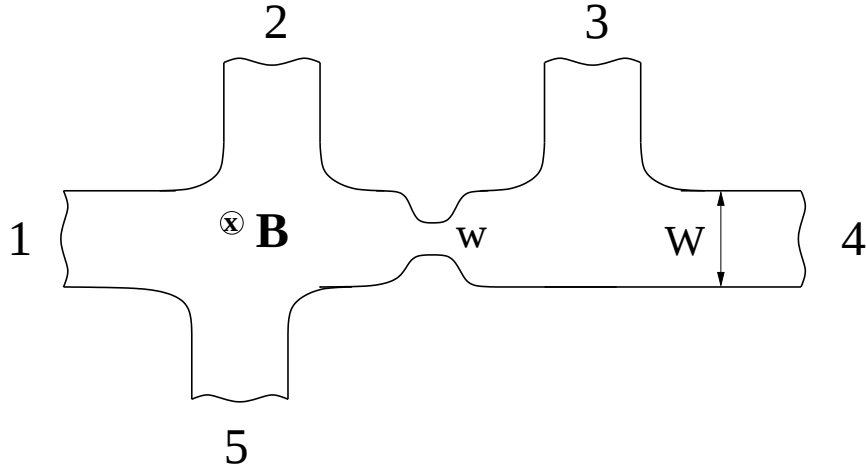
The figure shows the measured conductance of a narrow 1D channel (actually, a so-called "point contact"), where the width W of the channel is controlled by the voltage V_G on a split-gate electrode. The 2DEG "lives" in GaAs, so the effective electron mass is $m^* = 0.067m$, and the 2D density of electrons is $n_2 = 3.56 \cdot 10^{15} \text{ m}^{-2}$. Only the lowest 2D subband is occupied by electrons.

j) Use this information to calculate the Fermi level E_F in the 2DEG. (Hint: Use the result of 1g).)

k) Assume that the confining potential that defines the 1D channel is a potential box of width W and with hard walls (i.e. $V = \infty$ outside the 1D channel). With these assumptions, what is the width W of the 1D channel when the gate voltage is $V_G = -1.5 \text{ V}$?

QUESTION 2

Consider the 5-terminal device shown in the figure below, with ideal contacts, and with a geometrical constriction of width w defined by a split-gate technique. We ignore tunneling and impurity scattering.



A uniform perpendicular magnetic field $\mathbf{B}$ is pointing into the plane and causes the formation of N edge states (at each edge) in the wide regions of the device (width W). Inside the constriction, only n edge states (at each edge) are present at the Fermi level. Hence, we assume that n edge states are transmitted through the constriction with probability equal to one, whereas the remaining $N - n$ edge states do not enter the constriction. The Büttiker–Landauer equations,

$$I_{\alpha} = \sum_{\beta \neq \alpha} G_{\alpha\beta} (V_{\alpha} - V_{\beta}),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h} T_{\alpha\beta},$$

relate the current in terminal α to the potentials at the various terminals. Here, $T_{\alpha\beta}$ denotes the "transmission sum" from terminal β to terminal α . A small voltage is applied between terminals 1 and 4, resulting in a net current I flowing from terminal 1 to terminal 4 (i.e., $I_1 = -I_4 = I$). Terminals 2, 3, and 5 are used as voltage probes.

a) Write down the dimensionless 5×5 conductance matrix (or "transmission matrix") with matrix elements $T_{\alpha\beta} = hG_{\alpha\beta}/2e^2$.

b) Find the resistances $R_{14,23}$, $R_{14,25}$, $R_{14,35}$, and $R_{14,14}$. (Notation: $R_{\alpha\beta,\kappa\eta} = (V_{\kappa} - V_{\eta})/I$, $I = I_{\alpha} = -I_{\beta}$, $I_{\kappa} = I_{\eta} = 0$.) For convenience, choose $V_4 = 0$.

QUESTION 3

The figure below is copied from the paper *Weak Localization in Bilayer Graphene*, by R. B. Gorbachev et al (*Phys Rev Lett* **98**, 176805 (2007)):

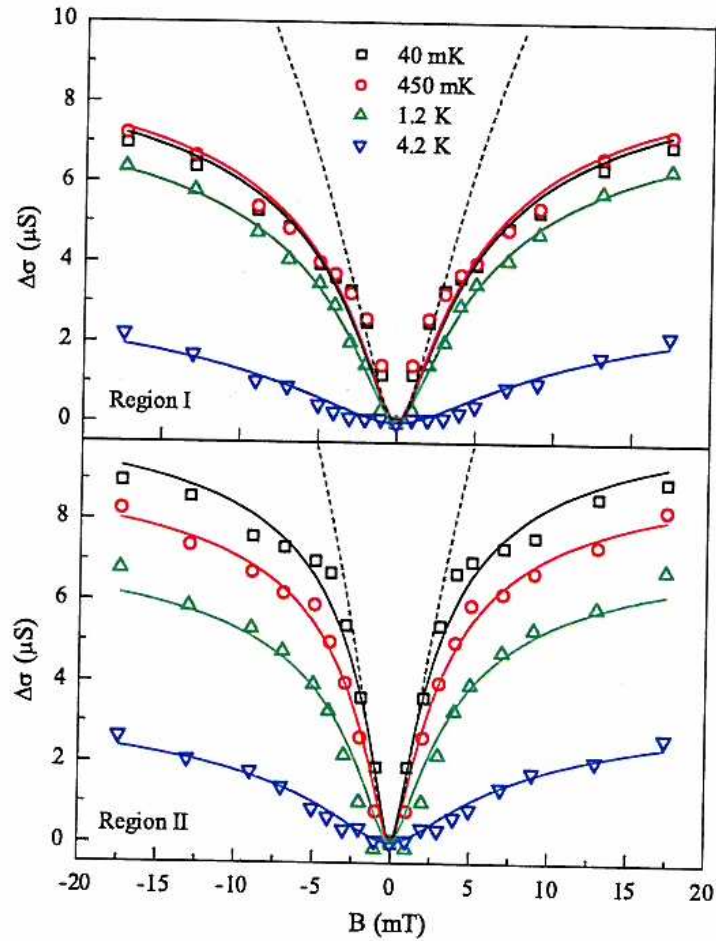


FIG. 3 (color online). Averaged magnetoconductivity for regions I and II. Dashed curves are the fits using only the first term in Eq. (1), and solid lines are the fits with the first two terms.

- Describe briefly, in a sentence or two, the physics behind "weak localization".
- Why does the conductivity increase when a weak magnetic field is applied to the sample?
- Why does the effect vanish with increasing temperature (as shown in the figure)?