

NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
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Contact during the exam:
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EXAM
TFY4340 MESOSCOPIC PHYSICS
Friday June 1 2012, 0900 - 1300
English

Remedies: C

- K. Rottmann: Mathematical formulae
- Calculator with empty memory

Pages 2 – 5: Problems 1 – 5. The five problems are relatively unrelated and may be answered in any order. Also, many of the subquestions within a given problem may be answered independently from the others.

Notation: Vectors are given in bold italic. Unit vectors are given with a hat above the symbol.

Some constants:

Electron mass: $m_e = 9.1 \cdot 10^{-31}$ kg. Elementary charge: $e = 1.6 \cdot 10^{-19}$ C.

Boltzmann constant: $k_B = 1.38 \cdot 10^{-23}$ J/K. Planck constant: $\hbar = h/2\pi = 1.05 \cdot 10^{-34}$ Js.

The grades will be available around June 10.

PROBLEM 1: Historical. [Weight: 10%. Suggested timing: 5 – 20 minutes.]

Connect Nobel laureate(s) with topic and year for receiving the prize:

- A Albert Einstein
- B Albert Fert/Peter Grünberg
- C Andre Geim/Konstantin Novoselov
- D Ivar Giaever/Leo Esaki/Brian Josephson
- E Klaus von Klitzing

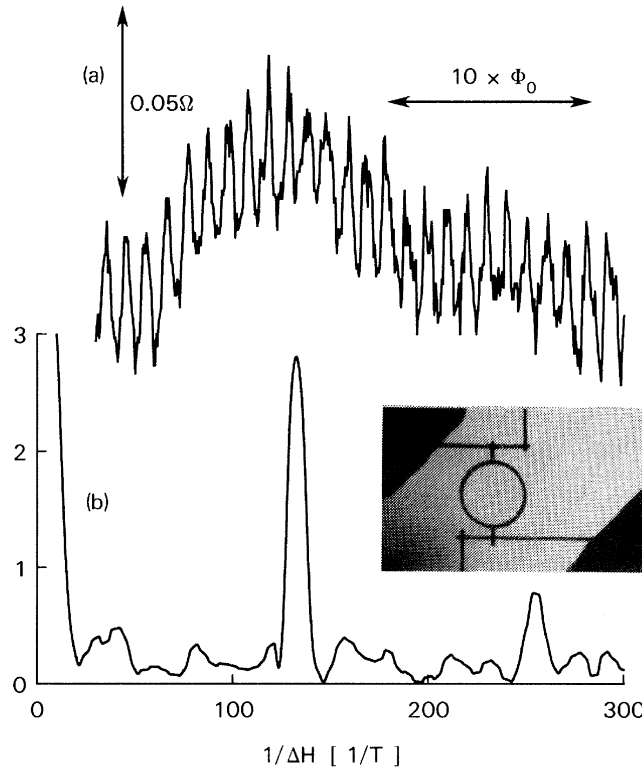
- 1 Giant magnetoresistance
- 2 Graphene
- 3 Photoelectric effect
- 4 Quantum Hall effect
- 5 Tunneling phenomena in semiconductors and superconductors

1921 1973 1985 2007 2010

PROBLEM 2: Short qualitative questions. [20%. 20 – 50 min.]

Use 1 – 3 sentences to answer each of the subquestions **a** – **e** below.

- a) Explain briefly the difference between Bloch states and surface states.
- b) To the question "What is mesoscopic physics?", what would your answer be?
- c) Assuming isotropic conditions (i.e., no directional dependence of relevant properties), how is the electron effective mass m^* defined in terms of the electronic band structure $E(k)$?
- d) How would you define, and hence distinguish between, a metal and an insulator, in terms of the electronic band structure?
- e) Explain briefly, with a figure and a few sentences, how the controlled layer-by-layer growth of GaAs and $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ can be used to construct a resonant tunneling device.

PROBLEM 3: Aharonov – Bohm effect.**[25%. 30 – 70 min.]**

The figure is taken from R. A. Webb et al, *Phys Rev Lett* **54**, 2696 (1985). Figure (a) displays the magnetoresistance of the metallic (gold) ring shown in the inset, i.e., $R(H)$, with H the (uniform) applied magnetic field perpendicular to the plane of the ring. (No distinction between H and B here!) The inside diameter of the ring is $d = 784 \text{ nm}$ and its wire width is $w = 41 \text{ nm}$. Figure (b) shows the Fourier spectrum of R , with the most prominent peak at $1/\Delta H = 131 \text{ T}^{-1}$. The temperature in the experiment was 10 mK .

a) It follows directly from Feynman's path integral approach to quantum mechanics that phase coherent electron transport in the presence of a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ manifests itself as a phase factor $\Delta\phi$ in the wavefunction,

$$\psi \sim \exp(i\Delta\phi) = \exp\left(-\frac{ie}{\hbar} \int \mathbf{A} \cdot d\mathbf{l}\right),$$

where the ("path") integral is taken along the path traversed by the electron. Show that quantum interference is responsible for the observed oscillatory behavior of the magnetoresistance $R(B)$ in the experiment by Webb et al. (*Aharonov - Bohm effect*.) In other words, show that the transmission probability T of the ring contains a B -dependent factor,

$$T \sim |\psi_T|^2 \sim 1 + \cos(2\pi B/B_0),$$

and find an expression for the "period" B_0 (in terms of fundamental constants and the ring geometry).

b) Verify that the peak in the Fourier spectrum of R at $1/B_0 = 1/\Delta H = 131 \text{ T}^{-1}$ is consistent with the size of the ring.

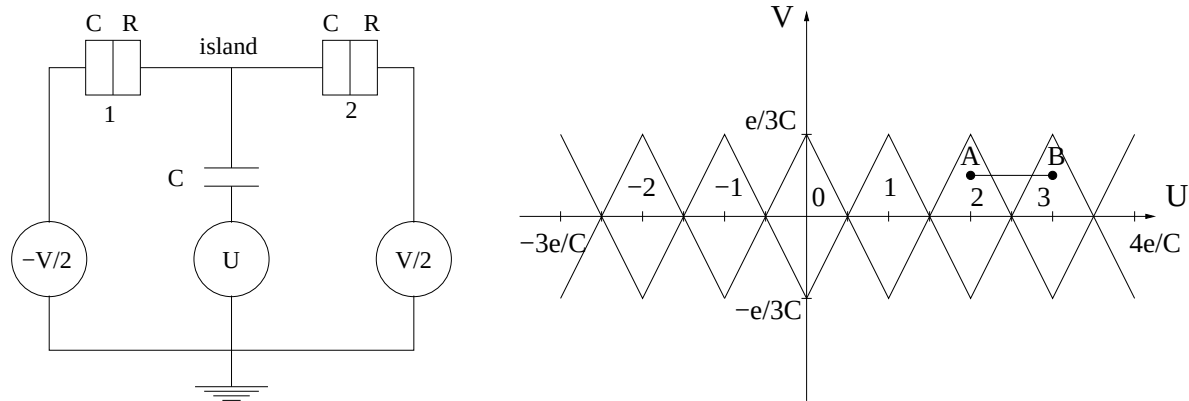
c) Explain briefly why the Aharonov - Bohm effect may be regarded as a *nonlocal* effect of the electromagnetic field. Explain also what is meant by *gauge invariance*, and argue that the Aharonov - Bohm effect is indeed gauge invariant.

d) In *The Big Bang Theory*, who is performing experiments related to the Aharonov - Bohm effect?

Given information: Stokes' theorem (for arbitrary vector field $\mathbf{G}$):

$$\int (\nabla \times \mathbf{G}) \cdot d\mathbf{S} = \oint \mathbf{G} \cdot d\mathbf{l}$$

PROBLEM 4: Single electron transistor. [15%. 10 – 30 min.]



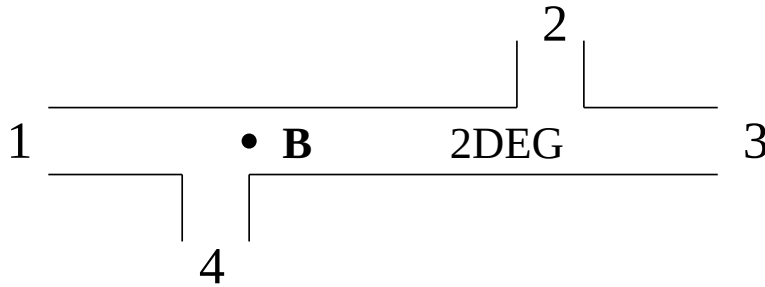
The single electron transistor shown in the left figure has a metallic island coupled to the outside world through two identical tunneling junctions 1 and 2, both with capacitance C and resistance R ($R \gg R_Q = h/e^2$), and an ideal capacitor C . The island has a net charge $q = -ne$, corresponding to n "extra" electrons. Constant bias voltages $\pm V/2$ are applied to the tunneling junctions, as illustrated in the figure. A time dependent gate voltage $U(t) = U_0 + U_1 \cos(2\pi ft)$ is connected to the ideal capacitor. The right figure shows the UV plane with diamond shaped areas, inside which a certain number of extra electrons (n) on the island corresponds to the equilibrium situation (i.e., the lowest energy). The appropriate value of n is indicated inside each diamond shaped area.

a) Assume a value of V corresponding to the horizontal line between A and B in the figure. Explain what happens when the gate voltage is set to oscillate with frequency f between the minimum value $U_0 - U_1$ at A and the maximum value $U_0 + U_1$ at B. Specifically, what is the resulting current I ?

b) If the system capacitance C is in the fF (femtofarad) range, at what temperature T should experiments be performed, in order to reveal clear single electron charging effects?

PROBLEM 5: Büttiker – Landauer and quantum Hall effect. [30%. 30 – 70 min.]

Consider the following ideal 4-terminal device:



A strong uniform magnetic field $\mathbf{B}$ is applied perpendicular to the 2DEG and points out of the plane.

The Büttiker–Landauer equations,

$$I_\alpha = \sum_{\beta \neq \alpha} G_{\alpha\beta} (V_\alpha - V_\beta),$$

with conductances

$$G_{\alpha\beta} = \frac{2e^2}{h} T_{\alpha\beta},$$

relate the net current entering into terminal α to the potentials at the various terminals. Here, $T_{\alpha\beta}$ denotes the "direct transmission sum" from terminal β to terminal α .

a) Assume that only the lowest Landau level lies below the Fermi energy E_F in the bulk region of the 2DEG. Express the relation between currents and potentials as

$$I_\alpha = \frac{2e^2}{h} \sum_{\beta=1}^4 \gamma_{\alpha\beta} V_\beta,$$

and write down the 4×4 matrix γ .

b) Let terminals 1 and 3 be the "source" and the "drain", respectively, whereas terminals 2 and 4 are ideal voltage probes. Find the Hall resistance $R_H = R_{13,24} = (V_2 - V_4)/I_1$ and the 2-terminal resistance $R_{2t} = R_{13,13} = (V_1 - V_3)/I_1$.

c) Interchange the roles of terminals 2 and 3 and find the 2-terminal resistance $R_{2t} = R_{12,12} = (V_1 - V_2)/I_1$ and the longitudinal resistance $R_L = R_{12,34} = (V_3 - V_4)/I_1$.

d) Qualitatively: With terminals 1 and 3 as source and drain, and terminals 2 and 4 as voltage probes (as in **b**)), explain how R_H will change as we decrease the magnetic field strength. Explain also how R_L depends on B when terminals 3 and 4 are used as voltage probes (as in **c**)).