NTNU

Department of Physics

Examination paper for TFY4345 Classical Mechanics

Academic contact during examination: Jaakko Akola

Phone: +358-452385276

Examination date: 11.12.2018

Examination time (from-to): 09:00-13:00

Permitted examination support material:

- Approved, simple calculator
- Fysiske størrelser og enheter: nav og symboler, Angell, Carl - Lian, Bjørn Ebbe
- K. Rottmann (matematisk formelsamling)
- Barnett & Cronin: Mathematical Formulae
- Notice: Supplementary Information on the last page of the exam contains useful formulas

Other information:

Grading: Problem 1 (6p), Problem 2 (6p), Problem 3 (6p), Problem 4 (6p), Problem 5 (6p) – Total 30 p

Language: English

Number of pages (front page excluded): 3

Number of pages enclosed: 3

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig □ **2-sidig** □

sort/hvit □ **farger** □

skal ha flervalgskjema □

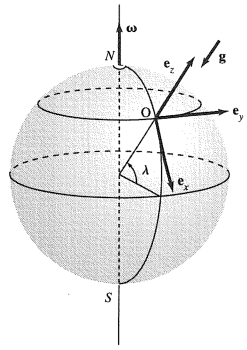
Checked by:

Date Signature

Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

Problem 1. Rotating coordinate system (6p = 20%)

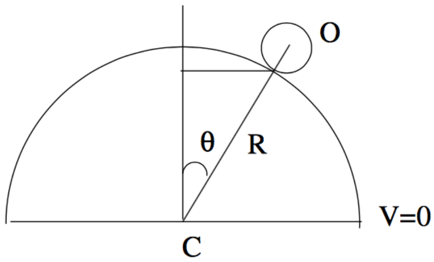
Tiril E. has adjusted her rifle sight for a 100-meter distance on a biathlon shooting station where the shooting direction is directly towards East (along $\mathbf{e}_y$-axis, see figure). The starting velocity of the rifle bullet is 600 m/s. Alas, the shooting track has been surprisingly reversed (?!) from East to West for the same distance in the competition, and Tiril does not have a chance to make any further sight adjustments.



How large is the resulting systematic deflection of hits due to this sudden change of conditions and in which direction? What about the effects of centripetal acceleration?

You can neglect the air resistance and assume a constant bullet velocity during the flight. The latitude of the shooting track is 60 degrees North (Holmenkollen). The angular velocity of Earth is 7.29×10^{-5} 1/s.

Problem 2. Hoop rolling down a sphere (6p = 20%)



A hoop of radius a is rolling down a cylinder without slipping. Let r be the distance from O to C. R is the radius of the cylinder, θ is the polar coordinate of O (center of hoop) and φ is the rotation angle of the hoop around its own axis.

(a) Determine the Lagrangian function and the two equations of constraints. What do the constraints describe and are they holonomic?
(b) Using Lagrange's undetermined multipliers, derive the equations of motion from Lagrange's equations.
(c) When does the hoop detach from the surface? [Hint: use here $\ddot{\theta} d\theta = \dot{\theta} d\dot{\theta}$]

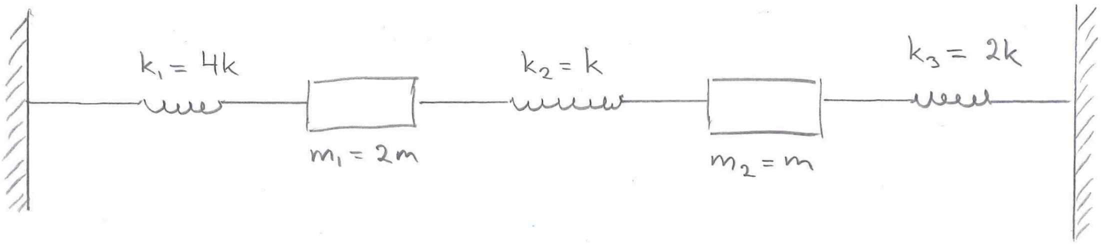
Problem 3. Lorentz transformation (6p = 20%)

Let us consider three inertial frames S_1, S_2 and S_3, with S_2 moving at a velocity $\beta = v/c$ in z-direction with respect to S_1 (coordinates x, y, z, t) and S_3 moving at a velocity $\beta' = v'/c$ in x'-direction with respect to S_2 (coordinates x', y', z', t'). Finally, an event in S_3 is denoted by coordinates x'', y'', z'', and t''.

(a) Derive the Lorentz transformation matrices for the two transformations $\bar{\bar{L}}$ and $\bar{\bar{L}}'$, where $\bar{x}' = \bar{\bar{L}} \cdot \bar{x}$ (from S_1 to S_2) and $\bar{x}'' = \bar{\bar{L}}' \cdot \bar{x}'$ (from S_2 to S_3).
(b) Derive the direct Lorentz transformation between S_1 and S_3, corresponding to the transformation $\bar{x}'' = \bar{\bar{L}}'' \cdot \bar{x}$. In other words, you should obtain the new coordinates (in S_3) as functions of the old ones (in S_1).

Problem 4. Coupled oscillations (6p = 20%)

Let us consider a system of two masses connected by springs to each other and walls on both sides, as shown in the figure below. The masses are $m_1 = 2m$ and $m_2 = m$, and the spring constants are $k_1 = 4k$, $k_2 = k$, and $k_3 = 2k$.



(a) Write out the Lagrangian function of the system and derive the secular determinant of coupled oscillations.
(b) Solve the eigenfrequencies and eigenvectors of oscillations.

Problem 5. Hamilton-Jacobi theory (6p = 20%)

Let us consider a system in free fall in a uniform gravitational field. The Hamiltonian of the system is

$$H = \frac{p^2}{2m} + mgy$$

where y is the vertical location and p is the associated canonical momentum.

In general, the Hamilton-Jacobi theory considers a partial differential equation of the form

$$H\left(q_i, \frac{\partial S}{\partial q_i}, t\right) + \frac{\partial S}{\partial t} = 0,$$

where S is the Hamilton's principal function.

(a) Derive the Hamilton's principal function $S(y, \alpha, t)$ and solve it. For simplicity, choose here that $\alpha = E$.
(b) By using $S(y, \alpha, t)$ and the interrelationships between old and new coordinates, derive the final solution for y. Use here the initial conditions $y = y_0$ and $p = p_0$ at $t = 0$.

Some useful formula and equations in the next page:

Cylindrical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

Spherical coordinates:

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Displacement in Minkowski-space:

$$ds^2 = dx_\mu dx_\mu = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2$$

Lorentz transformation:

$$\begin{aligned} L_{jk} &= \delta_{jk} + (\gamma - 1)\beta_j\beta_k/\beta^2 \\ L_{j4} &= i\gamma\beta_j \\ L_{4k} &= -i\gamma\beta_k \\ L_{44} &= \gamma \end{aligned}$$

Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}; \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Hamilton-Jacobi theory:

$$H + \frac{\partial S}{\partial t} = 0; \quad S = S(q_1, ..., q_n, \alpha_1, ..., \alpha_n, t)$$

$$p_i = \frac{\partial}{\partial q_i} S(q, \alpha, t); \quad Q_i = \frac{\partial}{\partial \alpha_i} S(q, \alpha, t) = \beta_i; \quad q_i = q_i(\alpha, \beta, t)$$

Observer in rotating coordinate system:

$$\vec{F}_{eff} = \vec{F} - 2m(\vec{\omega} \times \vec{v}_r) - m\vec{\omega} \times \vec{\omega} \times \vec{r}$$

Coupled oscillations:

$$V = \frac{1}{2}\sum_{j,k} A_{jk} q_j q_k; \quad T = \frac{1}{2}\sum_{j,k} m_{ij} \dot{q}_j \dot{q}_k$$