Problem 1. Set of questions (11p = 32%)

(a) Are the following statements true or false (T/F)? To obtain maximum scores you must explain your answers properly.

i. "The method of Lagrange's undetermined multipliers can only be used with holonomic constraints."

ii. "Two events whose 4-vector points outside the light cone can be causally connected."

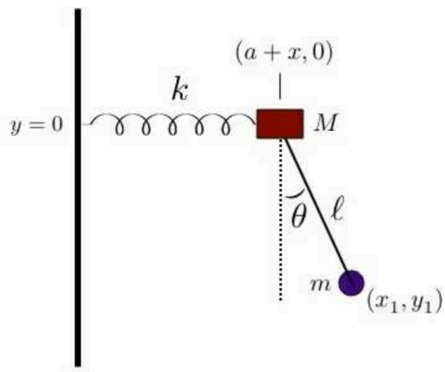
iii. "Kepler orbits are circular." [6p]

(b) Derive the answer for the following equation by starting from the formal definition of Poisson brackets: $[L_z, L_y]_{q,p} = ?$ [2p]

(c) Petter owns a Tesla car (m= 2200 kg). In a rush of endorphins and reckless behavior, he momentarily drives 250 km/h towards Værnes (East). The latitude is 63.25°. Calculate the direction and magnitude of the Coriolis force. [3p]

Problem 2. Pendulum attached on a harmonic oscillator (8p = 24%)

Consider next the system depicted in the figure right in which a mass M moves horizontally while attached to a spring of spring constant k. Hanging from this mass is a pendulum of arm length l and bob mass m.



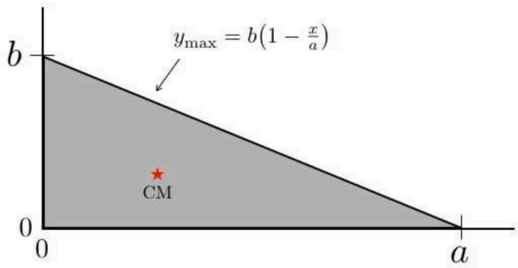
(a) Determine the Lagrangian function of the system by choosing appropriate generalized coordinates and derive the associated canonical momenta and canonical forces. [4p]

(b) Derive the equations of motion from the Lagrange's equations. [2p]

(c) Let us assume now that we are at the limit of small oscillations. Derive the simplified Lagrange's equations of motion by using the following definitions

$$u = \frac{x}{l},\ \alpha = \frac{m}{M},\ \omega_0^2 = \frac{k}{M},\ \omega_1^2 = \frac{g}{l}$$ [2p]

Problem 3. Principal moments of inertia of a triangular slab (9p = 26%)



(a) Compute the center-of-mass (COM) for the planar triangle in the figure right, assuming it to be of uniform two-dimensional mass density ρ. [2p]

(b) Compute the inertia tensor *with respect to the origin* for the same triangle. [3p]

(c) If the origin is shifted in the COM, the inertia tensor becomes (this can be shown by using the parallel axis theorem)

$$I^{COM} = \frac{M}{18}\begin{pmatrix} b^2 & \frac{1}{2}ab & 0 \\ \frac{1}{2}ab & a^2 & 0 \\ 0 & 0 & a^2+b^2 \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & 0 \\ I_{yx} & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

where $I_{xy} = I_{yx}$ and $I_{zz} = I_{xx} + I_{yy}$ in the general (latter) form. Define next

$$A = \tfrac{1}{2}\left(I_{xx} + I_{yy}\right)$$

$$B = \sqrt{\tfrac{1}{4}\left(I_{xx} - I_{yy}\right)^2 + I_{xy}{}^2}$$

$$\vartheta = \tan^{-1}\left(\frac{2I_{xy}}{I_{xx}-I_{yy}}\right)$$

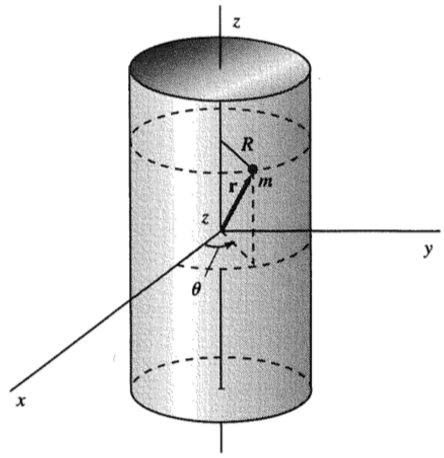
Derive the principal moments of inertia and principal axes of inertia by using the general form of the inertia tensor and these new variables. [4p]

Problem 4. Hamiltonian dynamics on a cylinder surface (6p = 18%)

Use the Hamiltonian method to find the equations of motion of a particle of mass m constrained to move on a cylinder surface defined by $x^2 + y^2 = R^2$. The particle is subject to a force directed towards the origin and squarely proportional to the distance from the origin: $F = -kr^2$.

(a) Derive the Hamiltonian function and the Hamiltonian equations of motion. [4p]

(b) Derive the final equation of motion for the z-coordinate. What can you say about conserved quantities? [2p]



Some useful formula and equations in the next page: