Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
Vår 2007

Løsningsforslag til øving 12

Veiledning uke 13

Oppgave 1

Kirchhoff's voltage rule (K2) yields

$$\mathcal{E} = \frac{Q}{C} = RI_R$$

whereas Kirchhoff's current rule (K1) yields

$$I = I_C + I_R$$

Furthermore, we have

$$I_C = \frac{dQ}{dt}$$

Hence:

$$\begin{aligned}
I_R(t) &= \frac{V_0}{R}\cos\omega t \\
Q(t) &= V_0 C\,\cos\omega t \\
I_C(t) &= -\omega C V_0\,\sin\omega t = \omega C V_0\,\cos(\omega t + \pi/2)
\end{aligned}$$

Total current delivered by the voltage source is then

$$I(t) = \frac{V_0}{R}\cos\omega t - \omega C V_0\,\sin\omega t$$

We want $I(t)$ on the form

$$I(t) = I_0\,\cos(\omega t - \alpha)$$

with amplitude $I_0 = V_0/Z$, where Z is the impedance of R and C coupled in parallel, whereas α becomes the phase angle, i.e., the phase difference between $\mathcal{E}(t)$ and $I(t)$. We have

$$\cos(\omega t - \alpha) = \cos\omega t\,\cos\alpha + \sin\omega t\,\sin\alpha$$

Hence, by direct comparison:

$$\begin{aligned}
\frac{\cos\alpha}{Z} &= \frac{1}{R} \\
\frac{\sin\alpha}{Z} &= -\omega C
\end{aligned}$$

These two equations, with two unknowns Z and α, are easily solved, and we find

$$\begin{aligned} Z &= \frac{R}{\sqrt{1+(\omega RC)^2}} \\ I_0 &= \frac{V_0}{R}\sqrt{1+(\omega RC)^2} \\ \alpha &= -\arctan(\omega RC) \end{aligned}$$

In the limit $\omega \to 0$ we should recover well known results from the DC examples given in the lectures, and so we do: $Z \to R$ and $\alpha \to 0$ so that $I_0 \to V_0/R$. All the current runs through the resistor R, while the capacitor C now represents an open circuit, where no direct current can run.

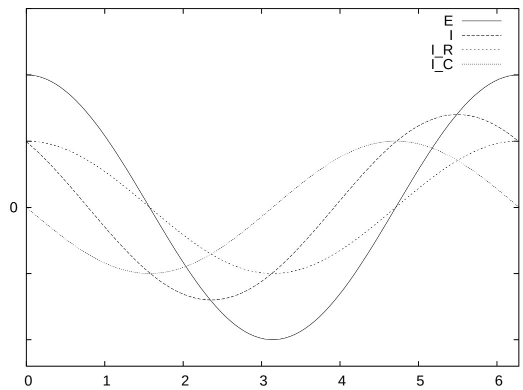
With given numerical values, we have

$$\omega RC = 2\pi \cdot 10^6 \cdot 10 \cdot 16 \cdot 10^{-9} = 1.0$$

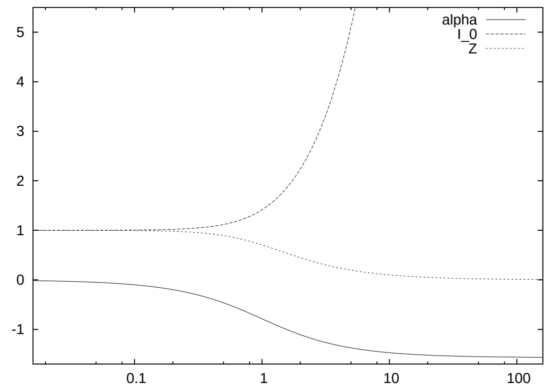
so that

$$\begin{aligned} I_0 &= \frac{1.0}{10} \cdot \sqrt{2} = 0.14 \text{ A} \\ \alpha &= -\arctan 1.0 = -45^\circ \end{aligned}$$

Sketch of $\mathcal{E}(t)$, $I(t)$, $I_R(t)$ and $I_C(t)$:



Sketch of α, I_0 and Z (with α in radians and ωRC between 0.016 and 160 along the horizontal axis):


alpha
I_0
Z
5
4
3
2
1
0
-1
0.1
1
10
100