**Institutt for fysikk, NTNU**
**TFY4155/FY1003 Elektrisitet og magnetisme**
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# Løsningsforslag til øving 13

Veiledning uke 15.

*Oppgave 2*

*a*) Distance vector (from the origin) to a point in the ring:

$$\boldsymbol{r} = s\hat{s} + z\hat{z}$$

Current density is:

$$\boldsymbol{j} = \frac{I}{ab}\hat{\phi}$$

Hence:

$$\boldsymbol{r} \times \boldsymbol{j} = \frac{I}{ab}\left(s\hat{z} - z\hat{s}\right)$$

Magnetic dipole moment becomes:

$$\boldsymbol{m} = \frac{I}{2ab}\int \left(s\hat{z} - z\hat{s}\right) s\, ds\, dz\, d\phi$$

where $\phi$ runs from 0 to $2\pi$, $s$ from $R - a/2$ to $R + a/2$ and $z$ from $z_0$ to $z_0 + b$. In the first term, i.e., the one which is proportional with $\hat{z}$, integration over $z$ gives a factor $b$ while the integration over $\phi$ gives a factor $2\pi$. The integral over $s$ becomes

$$\int_{R-a/2}^{R+a/2} s^2 ds = \frac{1}{3}\left(R + a/2\right)^3 - \frac{1}{3}\left(R - a/2\right)^3$$

The second term, i.e., the one which is proportional with $\hat{s}$, must, by symmetry arguments, disappear. We may also show this explicitly: The unit vector $\hat{s}$ is *not* a constant, but depends on the angle $\phi$ in the following manner:

$$\hat{s} = (\cos\phi)\ \hat{x} + (\sin\phi)\ \hat{y}$$

Hence, this term will be proportional with

$$\int_0^{2\pi} \hat{s}\, d\phi = \int_0^{2\pi} \left(\cos\phi\ \hat{x} + \sin\phi\ \hat{y}\right) d\phi = 0$$

So, the magnetic dipole moment of the ring is:

$$\begin{aligned}
\boldsymbol{m} &= \frac{I}{2ab} \cdot b \cdot 2\pi \cdot \left[\frac{1}{3}\left(R + a/2\right)^3 - \frac{1}{3}\left(R - a/2\right)^3\right] \hat{z} \\
&= \frac{I\pi R^3}{3a}\left[\left(1 + \frac{a}{2R}\right)^3 - \left(1 - \frac{a}{2R}\right)^3\right] \hat{z}
\end{aligned}$$

*b*) With a thin ring, $a \ll R$, this expression for $\boldsymbol{m}$ may be expanded to lowest order in the ratio $a/R$:

$$\left(1 + \frac{a}{2R}\right)^3 \simeq 1 + \frac{3a}{2R}$$
$$\left(1 - \frac{a}{2R}\right)^3 \simeq 1 - \frac{3a}{2R}$$

Then, one has

$$\boldsymbol{m} \simeq I\pi R^2 \hat{z}$$

Alternatively, you may simply multiply out the two parentheses that are raised to the third power. This becomes no more than 4 terms from each. Then you will find:

$$\left(1 + \frac{a}{2R}\right)^3 - \left(1 - \frac{a}{2R}\right)^3 = \frac{3a}{R} + \frac{a^3}{4R^3} \simeq \frac{3a}{R}$$

where the final approximation could be performed because $a \ll R$.