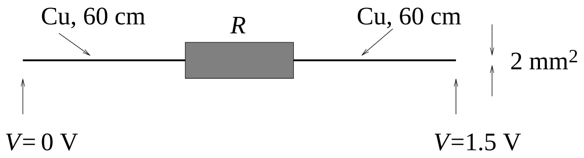
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2008

Øving 11

Veiledning: Fredag 28. og mandag 31. mars
Innleveringsfrist: Fredag 4. april

Exercise 1

A voltage source $V = 1.5$ V is connected to a resistor with resistance $R = 20\ \Omega$ via two 60 cm long copper wires with cross section 2 mm^2.



a) How big is the voltage drop over the Cu wires and the resistor, respectively? [Answers: 0.75 mV and 1.5 V]

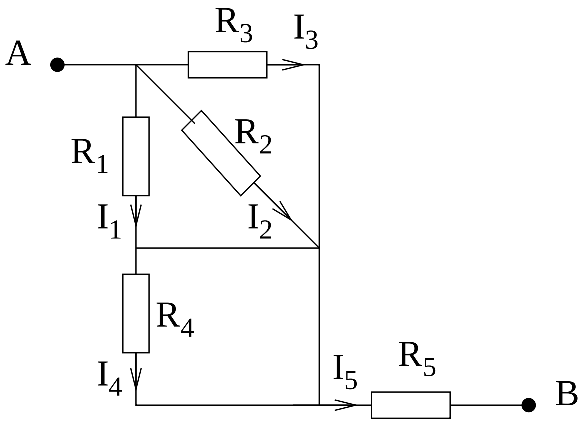
b) Determine the current and the power dissipated (”lost”) in the resistor. [Answers: ca 0.075 A and 0.11 W]

c) What is the average drift velocity of the free electrons through the Cu wires? Assume one free electron from each Cu atom. Compare with the average thermal velocity for an electron at room temperature. (Average kinetic energy pr electron at temperature T is $3k_BT/2$, where k_B is Boltzmann's constant.)
[Answers: 2.76 μm/s and ca 10^5 m/s.]

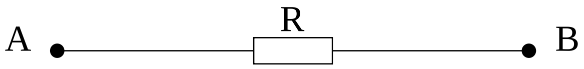
Given information: Mass density of Cu: 8960 kg/m^3. Molar mass of Cu: 63.54 g/mol. Electric conductivity for Cu at room temperature: $5.8 \cdot 10^7\ \Omega^{-1}\text{m}^{-1}$. Boltzmann's constant: $k_B = 1.38 \cdot 10^{-23}$ J/K.

Exercise 2

The figure below shows an electric circuit with 5 resistors R_j, $j = 1, .., 5$.



a) Determine the total resistance R between the points A and B, i.e.: Determine the resistance R in the equivalent circuit in the following figure:



b) An ideal voltage source with electromotive force $\mathcal{E}$ is connected to the circuit so that $\Delta V = V_A - V_B = \mathcal{E}$. Determine the resulting currents I_j through each of the resistors R_j.
(Unless otherwise specified, we always assume, in exercises like this one, that the connecting wires between the various resistors are *perfect conductors*, i.e., with zero resistance.)

Exercise 3

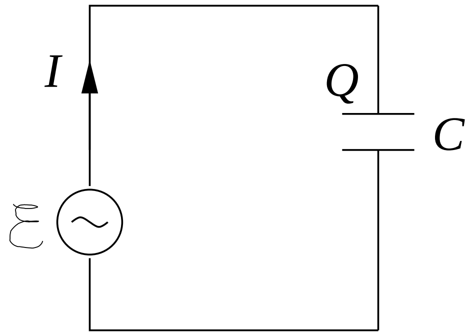
A capacitor with capacitance C has charge $\pm Q_0$. The capacitor is at time $t = 0$ connected to a resistor R, so that we have a closed circuit, and current may run in the circuit. Determine the capacitor charge $Q(t)$ and the current $I(t)$ for $t \geq 0$.

Exercise 4

For a DC circuit with voltage source V_0 delivering a current I_0, the total resistance R of the circuit will be given by Ohm's law, $R = V_0/I_0$.
For an AC circuit with voltage source $V_0 \cos \omega t$ delivering a current $I_0 \cos(\omega t - \alpha)$, we may analogously define a "generalized resistance", or impedance Z, for the circuit: $Z = V_0/I_0$. We allow for a *phase difference* α between applied voltage and resulting current. We will also see, in the examples below, that the current amplitude I_0 may become dependent upon the (angular) frequency ω, which implies that the impedance Z may also be frequency dependent,

i.e., $Z = Z(\omega)$.
In the lectures, I did the simplest example, with a voltage source $V_0 \cos\omega t$ connected to a single resistor R, and found that the current becomes $I_0 \cos\omega t$, with amplitude $I_0 = V_0/R$. So, the impedance for a resistor is simply $Z_R = R$.

a) A voltage source $\mathcal{E}(t) = V_0 \cos\omega t$ is connected to a capacitor C:

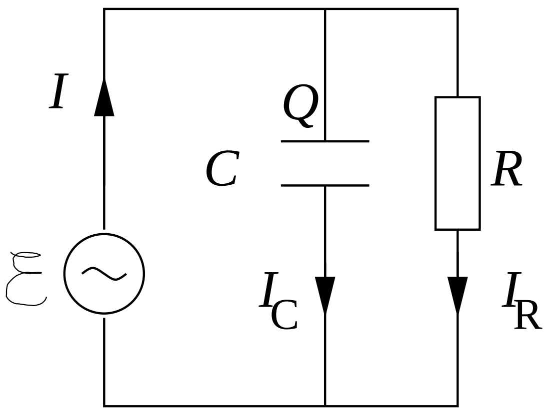


Find the charge $Q(t)$ on the capacitor, and also the current $I(t)$ in the circuit. Sketch $\mathcal{E}(t)$ and $I(t)$ between $t = 0$ and $t = T = 2\pi/\omega$.
Show that the current may be written in the form

$$I(t) = I_0 \cos(\omega t - \alpha)$$

and determine (the frequency dependent) current amplitude I_0 and the phase constant α. What is then the impedance $Z_C(\omega)$ of a capacitor C?

b) A voltage source $\mathcal{E}(t) = V_0 \cos\omega t$ is connected to a capacitor C and a resistor R connected in parallel:



Use Kirchhoff's rules to find the currents $I(t)$, $I_C(t)$ and $I_R(t)$. Show that the total current delivered by the voltage source may be written as

$$I(t) = I_0 \cos(\omega t - \alpha)$$

with amplitude

$$I_0 = \frac{V_0}{R} \sqrt{1 + (\omega RC)^2}$$

and phase constant

$$\alpha = -\arctan(\omega RC)$$

Hint: Use $\cos(a \pm b) = \cos a \, \cos b \mp \sin a \, \sin b$.

Assume the voltage source has amplitude $V_0 = 1.0$ V and frequency $f = 1.0$ MHz, and also assume $R = 10\,\Omega$ and $C = 16$ nF. Determine numerical values for I_0 and α. Sketch $\mathcal{E}(t)$, $I(t)$, $I_R(t)$ and $I_C(t)$ for t between 0 and $T = 1/f$. (Answers: $I_0 = 0.14$ A, $\alpha = -45°$)

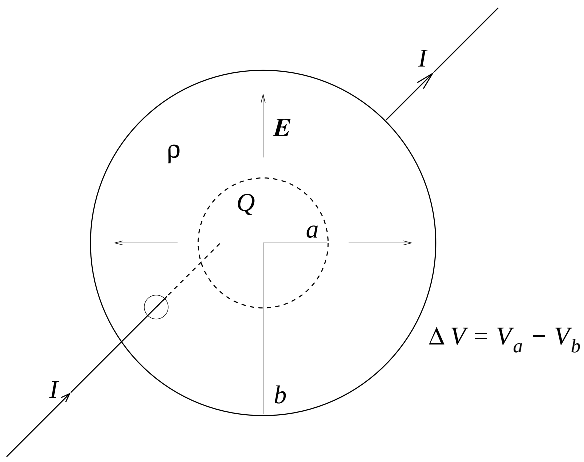
With the given numerical values of V_0, R and C, sketch the functions $I_0(\omega)$, $\alpha(\omega)$, and the impedance of the circuit, $Z(\omega) = V_0/I_0(\omega)$ for angular frequencies between 10^5 and 10^9 s^{-1}. Hint: Use a *logarithmic* scale along the ω axis, i.e., sketch I_0 and α for angular frequencies so that $\log \omega$ varies between the values 5 and 9.

Exercise 5

The figure shows two spherical conductors with radius a (the inner sphere) and b (the outer sphere), respectively. The region between the conducting spheres is filled with a material with resistivity ρ.
(Note: Here, the symbol ρ denotes resistivity, or inverse conductivity, since $\rho = 1/\sigma$. So here, ρ does not mean charge pr unit volumd...!)
A thin, isolated conducting wire passes through a little hole in the outer spherical conductor and in to the inner sphere. A stationary (i.e., time independent) electric current runs "through the system" as shown in the figure. Then, the potential difference between the inner and outer sphere is $\Delta V = V_a - V_b$, with the largest value of the potential on the inner sphere. Assume that the connecting wires has a negligible resistance in comparison with the material between the two spheres. Show that the resistance of this system is $R = \rho(a^{-1} - b^{-1})/4\pi$. You may do this in one of two ways (or both, if you like!):

1. Start with the resistance of a thin spherical shell with radius r and thickness dr, which is $dR = \rho dr/4\pi r^2$.

2. Start by assuming that the inner sphere has charge Q, and determine the two quantities ΔV and the current $I = \int \boldsymbol{j} \cdot d\boldsymbol{A} = \rho^{-1} \int \boldsymbol{E} \cdot d\boldsymbol{A}$. (So you may use Gauss' law...!)


I
E
ρ
Q
a
ΔV = V_a − V_b
I
b