Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
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Solution to øving 11

Guidance March 31 and April 1

Exercise 1

a) At first, we should try to realize that what we have here is the following circuit: [a parallel connection of R_1, R_2 and R_3] coupled in series with [a parallel connection of R_4 and $R_0 = 0$] in series with [R_5]. In other words, the resistance R_4 is "cut short", so that no current passes through R_4. (Alternatively: We have the same value for the potential on each side of R_4. Then, no current can pass through it.) Thus, the total resistance is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} + R_5$$

b) It should be clear that the total current I in the circuit must be the same as the current I_5 passing through R_5. Further, it should also be clear that I must distribute itself on the three currents passing through R_1, R_2 and R_3: $I = I_1 + I_2 + I_3$. In *a*) above, we have already concluded that no current passes through R_4: $I_4 = 0$.
The voltage drop across the three upper resistances is the same:

$$V' = R_1 I_1 = R_2 I_2 = R_3 I_3$$

The voltage drop across R_5 is

$$V'' = R_5 I_5 = R_5 I = R_5 \frac{\mathcal{E}}{R}$$

These two together must equal the value of the voltage source:

$$\mathcal{E} = V' + V''$$

Thus

$$V' = \mathcal{E} - V'' = \mathcal{E} - R_5 \frac{\mathcal{E}}{R} = \mathcal{E}\left(1 - \frac{R_5}{R}\right)$$

And finally,

$$I_1 = \frac{V'}{R_1}$$

$$I_2 = \frac{V'}{R_2}$$

$$I_3 = \frac{V'}{R_3}$$

c) With the given numerical values, we have

$$R = \left(1 + \frac{1}{2} + \frac{1}{3}\right)^{-1} + 5 = \frac{61}{11}\ \Omega$$

Thus,

$$V' = 9 \cdot \left(1 - \frac{5}{61/11}\right) = \frac{54}{61}\ \mathrm{V}$$

and

$$V'' = 9 - \frac{54}{61} = \frac{495}{61}\ \mathrm{V}$$

The various currents are

$$I_1 = \frac{54}{61 \cdot 1} = \frac{54}{61}\ \mathrm{A} \simeq 0.885\ \mathrm{A}$$
$$I_2 = \frac{54}{61 \cdot 2} = \frac{27}{61}\ \mathrm{A} \simeq 0.443\ \mathrm{A}$$
$$I_3 = \frac{54}{61 \cdot 3} = \frac{18}{61}\ \mathrm{A} \simeq 0.295\ \mathrm{A}$$
$$I_5 = I = \frac{9 \cdot 11}{61} = \frac{99}{61}\ \mathrm{A} \simeq 1.623\ \mathrm{A}$$

Exercise 2

a) Bulb 1 will be brightest in circuit B and weakest in circuit A. In circuit A, the voltage drop across bulb 1 is only 1/3 of the emf of the applied voltage source. In B, the voltage drop across all the bulbs is equal to the emf of the applied voltage source. In C, the parallel connection of 2 and 3 constitutes a resistance $(1/R + 1/R)^{-1} = R/2$, if the resistance in one bulb is R. Then, the voltage drop over bulb 1 becomes 2/3 of the emf of the applied voltage source, and the light intensity somewhere between that in A and B.

b) In A, we end up with an open ("broken") circuit, and therefore zero current, i.e., bulbs 1 (and 2) go out. In B, bulb 3 does not at all influence the voltage across bulb 1, so the light intensity is unchanged. In C, we end up with two resistances R coupled in series, so the voltage drop across bulb 1 must be half of the emf of the applied voltage source. I.e., smaller than what it was when bulb 3 was in place. Thus, the light intensity becomes smaller in circuit C.

Exercise 3

Let us put time $t = 0$ when the student cuts the circuit in A. Before this, we have a stationary situation, with voltage drop

$$V_0 = V_R = V_C$$

over both the resistance R and the capacitance C. At $t = 0$, the circuit is broken ("opened") in A, and we are left with the loop with R and C (and no voltage source anymore). As derived in the lectures, we now get a discharge of the capacitor, with a time dependence

$$Q(t) = Q_0 e^{-t/RC}$$

where

$$Q_0 = Q(0) = CV_0$$

After a certain time t_1, the circuit is broken at B, which means that the discharge stops. Then we have a charge

$$Q(t_1) = Q_0 e^{-t_1/RC}$$

on the capacitor. This corresponds to a voltage drop

$$V(t_1) = \frac{Q(t_1)}{C} = \frac{Q_0}{C} e^{-t_1/RC}$$

across the capacitor. During this time, the student has moved a distance d, so the velocity must be

$$v = d/t_1$$

We solve for t_1 in the expression for $V(t_1)$:

$$t_1 = RC \ln \frac{V_0}{V(t_1)}$$

where we have used $Q_0 = V_0 C$ and $\ln x = -\ln(1/x)$. With numerical values inserted:

$$t_1 = 150 \cdot 10^{-3} \cdot \ln \frac{9.00}{3.58} = 0.138 \text{ s}$$

The velocity becomes

$$v = d/t_1 = 1.00/0.138 = 7.23 \text{ m/s} \simeq 26 \, \text{km/h}$$