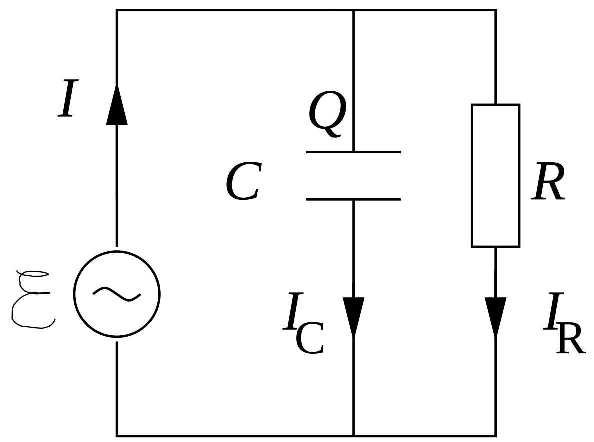
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2007

Øving 12

Veiledning: Uke 13
Innleveringsfrist: Onsdag 11. april

Exercise 1

An AC voltage source $\mathcal{E}(t) = V_0 \cos\omega t$ is connected to a circuit consisting of a capacitance C and a resistance R in parallel:



Use Kirchhoff's rules to find the currents $I(t)$, $I_C(t)$ and $I_R(t)$. Show that the total current delivered by the voltage source may be written

$$I(t) = I_0 \cos(\omega t - \alpha)$$

with amplitude

$$I_0 = \frac{V_0}{R}\sqrt{1 + (\omega RC)^2}$$

and phase constant

$$\alpha = -\arctan(\omega RC)$$

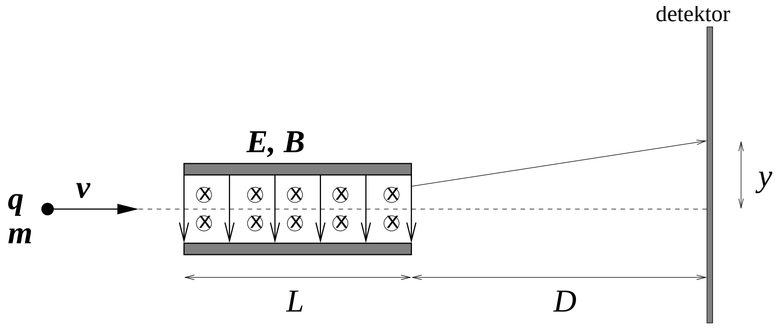
Hint: Use $\cos(a \pm b) = \cos a \, \cos b \mp \sin a \, \sin b$.

Assume that the voltage source has amplitude $V_0 = 1.0$ V and frequency $f = 1.0$ MHz, and that $R = 10\,\Omega$ and $C = 16$ nF. Find numerical values for I_0 and α. Sketch $\mathcal{E}(t)$, $I(t)$, $I_R(t)$ and $I_C(t)$ for t between 0 and $1/f$.

With the given values of V_0, R and C, sketch the functions $I_0(\omega)$, $\alpha(\omega)$ and the *impedance* of the circuit, $Z(\omega) = V_0/I_0(\omega)$ for angular frequencies between 10^5 and 10^9 s^{-1}. Hint: Use a logarithmic scale along the ω axis.

Exercise 2

Particles with mass m, charge q, and velocity $\boldsymbol{v}$ enter a region with "crossed" electric and magnetic field, $\boldsymbol{E}$ and $\boldsymbol{B}$, as shown in the figure. $\boldsymbol{E}$ is directed downwards, and $\boldsymbol{B}$ is directed into the paper plane. In the region with extent L we assume that the fields are homogeneous. Outside this region, $\boldsymbol{E} = \boldsymbol{B} = 0$.



You keep the electric field strength E constant throughout the experiment. First, you set $B = 0$ and register that the particles are deflected and hit the detector in a distance y above the center line (which is dashed). Next, you repeat the experiment, but this time you adjust the value of B until the particles are no longer deflected.
Show that the ratio between the charge and the mass of the particles is proportional to the deflection (when $B = 0$), i.e.,

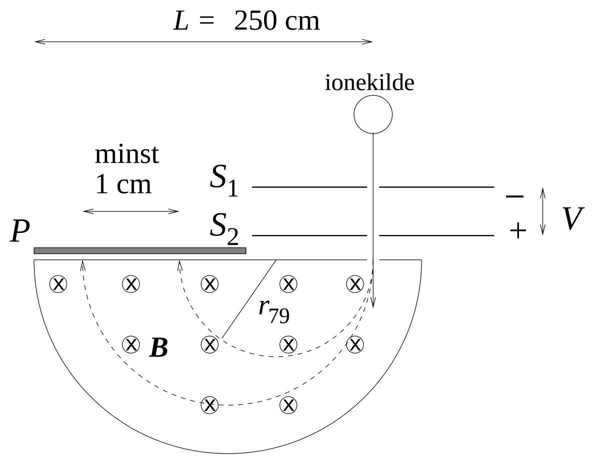
$$\frac{q}{m} = -ay,$$

and find a expressed in terms of E, B, D, and L.

Given information: $\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ (the Lorentz force)

In this manner, J. J. Thomson analyzed so called cathode rays in 1897, and showed that these rays consisted of a particular type of particles with negative charge. These were electrons that were emitted from the metal of the cathode. Thomson was the first one to determine the ratio e/m_e. He found the same value for this ratio, independent of what kind of metal he used in the cathode. Thus, he could conclude that the observed particles had to be a *fundamental* ingredient of nature.

Exercise 3



The figure shows a mass spectrometer. An ion source emits charged particles. Two slits, S_1 and S_2, make sure that a well *collimated* beam of particles enter into the region with the magnetic field $\boldsymbol{B}$ (which is directed into the paper plane). Between S_1 and S_2, we have a voltage difference V which accelerates the ions. The particle velocity at S_2 is much larger than at S_1, so we may put $v = 0$ at S_1. The ions are deflected through an angle 180° by the magnetic field and are detected on a photographic plate P.
The spectrometer is supposed to be used to separate the bromine isotopes ^{79}Br and ^{81}Br. The source emits these isotopes as ions with charge $-e$. You may assume that the isotopes have atomic masses $79m_p$ and $81m_p$, respectively.
On the photographic plate, we want a separation of at least 1.0 cm between the points where the two isotopes strike. At the same time, we must make sure that both isotopes really do hit the plate, which has a width $L = 250$ cm, measured from where the ions enter into the magnetic field. With these two conditions, what is the upper and lower limit for the strength of the magnetic field B, when the accelerating voltage drop V is 400 V?

Given information: $e = 1.6 \cdot 10^{-19}$ C, $m_p = 1.67 \cdot 10^{-27}$ kg
[One of the answers is: $B_{\text{min}} = 21$ mT]