Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
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Løsningsforslag til øving 13

Veiledning uke 15.

Oppgave 2

a) Distance vector (from the origin) to a point in the ring:

$$\boldsymbol{r} = s\hat{s} + z\hat{z}$$

Current density is:

$$\boldsymbol{j} = \frac{I}{ab}\hat{\phi}$$

Hence:

$$\boldsymbol{r} \times \boldsymbol{j} = \frac{I}{ab}\left(s\hat{z} - z\hat{s}\right)$$

Magnetic dipole moment becomes:

$$\boldsymbol{m} = \frac{I}{2ab}\int \left(s\hat{z} - z\hat{s}\right) s\, ds\, dz\, d\phi$$

where ϕ runs from 0 to 2π, s from $R - a/2$ to $R + a/2$ and z from z_0 to $z_0 + b$. In the first term, i.e., the one which is proportional with $\hat{z}$, integration over z gives a factor b while the integration over ϕ gives a factor 2π. The integral over s becomes

$$\int_{R-a/2}^{R+a/2} s^2 ds = \frac{1}{3}\left(R + a/2\right)^3 - \frac{1}{3}\left(R - a/2\right)^3$$

The second term, i.e., the one which is proportional with $\hat{s}$, must, by symmetry arguments, disappear. We may also show this explicitly: The unit vector $\hat{s}$ is *not* a constant, but depends on the angle ϕ in the following manner:

$$\hat{s} = (\cos\phi)\ \hat{x} + (\sin\phi)\ \hat{y}$$

Hence, this term will be proportional with

$$\int_0^{2\pi} \hat{s}\, d\phi = \int_0^{2\pi} \left(\cos\phi\, \hat{x} + \sin\phi\, \hat{y}\right) d\phi = 0$$

So, the magnetic dipole moment of the ring is:

$$\begin{aligned}
\boldsymbol{m} &= \frac{I}{2ab}\cdot b \cdot 2\pi \cdot \left[\frac{1}{3}\left(R + a/2\right)^3 - \frac{1}{3}\left(R - a/2\right)^3\right]\hat{z} \\
&= \frac{I\pi R^3}{3a}\left[\left(1 + \frac{a}{2R}\right)^3 - \left(1 - \frac{a}{2R}\right)^3\right]\hat{z}
\end{aligned}$$

b) With a thin ring, $a \ll R$, this expression for $\boldsymbol{m}$ may be expanded to lowest order in the ratio a/R:

$$\left(1+\frac{a}{2R}\right)^3 \simeq 1+\frac{3a}{2R}$$
$$\left(1-\frac{a}{2R}\right)^3 \simeq 1-\frac{3a}{2R}$$

Then, one has

$$\boldsymbol{m} \simeq I\pi R^2 \hat{z}$$

Alternatively, you may simply multiply out the two parentheses that are raised to the third power. This becomes no more than 4 terms from each. Then you will find:

$$\left(1+\frac{a}{2R}\right)^3 - \left(1-\frac{a}{2R}\right)^3 = \frac{3a}{R} + \frac{a^3}{4R^3} \simeq \frac{3a}{R}$$

where the final approximation could be performed because $a \ll R$.