

Formulas TFY4345

Validity of formulas and meaning of symbols are assumed to be known.

- Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad , \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

- Lagrange's equations:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

- Poisson brackets:

$$[f, g] = \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial g}{\partial q_i} \frac{\partial f}{\partial p_i}$$

$$[q_i, q_j] = [p_i, p_j] = 0 \quad , \quad [q_i, p_j] = \delta_{ij}$$

- Four vector:

$$x_\mu = (\mathbf{r}, i c t)$$

- Four potential:

$$A_\mu = (\mathbf{A}, i \phi/c)$$

- Electromagnetic field:

$$\begin{aligned} \mathbf{E} &= -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \nabla \times \mathbf{A} \end{aligned}$$

- Lorentz transformation (with relative velocity $\mathbf{v} = v\hat{x}$):

$$L_{22} = L_{33} = 1 \quad , \quad L_{11} = L_{44} = \gamma \quad , \quad L_{14} = -L_{41} = i\beta\gamma$$

$$\beta = v/c \quad , \quad \gamma = 1/\sqrt{1 - v^2/c^2}$$

- Lorentz transformation of electromagnetic field (where the inertial system S_0 moves with velocity $v\hat{x}$ with respect to S):

$$\begin{aligned} E_x &= E_{0x} \quad , \quad E_y = \gamma(E_{0y} + vB_{0z}) \quad , \quad E_z = \gamma(E_{0z} - vB_{0y}) \\ B_x &= B_{0x} \quad , \quad B_y = \gamma(B_{0y} - \frac{v}{c^2}E_{0z}) \quad , \quad B_z = \gamma(B_{0z} + \frac{v}{c^2}E_{0y}) \end{aligned}$$

- Trigonometric relations:

$$\begin{aligned} \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \end{aligned}$$