TFY4345 Classical Mechanics. Department of Physics, NTNU.

ASSIGNMENT 2

Question 1

Use Hamilton's principle to show that the transformed Lagrangian

$$L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{dF(q, t)}{dt},$$

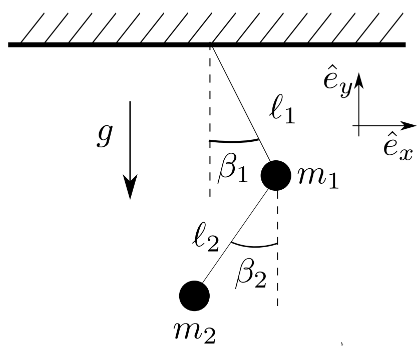
where F is an arbitrary function of q and t, leads to the same equations of motion (the Lagrange equations) as the original Lagrangian $L(q, \dot{q}, t)$. Hint: No variation in the end points.

Question 2

In Assignment 1, we derived the equations of motion for the double pendulum (see figure below):

$$\begin{aligned}
(m_1 + m_2)\ell_1^2\ddot{\beta}_1 + m_2\ell_1\ell_2\ddot{\beta}_2\cos(\beta_1 - \beta_2) + m_2\ell_1\ell_2\dot{\beta}_2^2\sin(\beta_1 - \beta_2) + (m_1 + m_2)g\ell_1\sin\beta_1 &= 0 \\
m_2\ell_2^2\ddot{\beta}_2 + m_2\ell_1\ell_2\ddot{\beta}_1\cos(\beta_1 - \beta_2) - m_2\ell_1\ell_2\dot{\beta}_1^2\sin(\beta_1 - \beta_2) + m_2 g\ell_2\sin\beta_2 &= 0
\end{aligned}$$

Let $m_1 = m_2 = m$, $\ell_1 = \ell_2 = \ell$, linearize the equations, use the ansatz $\beta_i = A_i\cos\omega t$ $(i = 1, 2)$ with constant amplitudes A_i, and find the two possible values of the frequency ω. Finally, find the ratio A_1/A_2 (including sign) for each oscillation mode.



Question 3

Use the Levi-Civita tensor and derive the following relations:

$$\begin{aligned}
\nabla \times (\nabla \times \boldsymbol{A}) &= \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} \\
\boldsymbol{V} \times (\nabla \times \boldsymbol{V}) &= \frac{1}{2}\nabla V^2 - (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} \\
\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) &= \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B})
\end{aligned}$$

Question 4

Show that the shortest distance between two points in the plane is a straight line. Use cartesian coordinates (x, y) or polar coordinates (r, θ) – or do it both ways.