**TFY4345 Classical Mechanics. Department of Physics, NTNU.**

**ASSIGNMENT 5**

## Question 1

a) Write a program that produces the curve

$$r = \frac{p}{1 + \varepsilon \cos(\gamma\theta)}.$$

Choose (e.g.) $p = 1$, and a rather small value of $\gamma$, and let the angle $\theta$ vary from 0 to $\pi/2\gamma$ when you plot the curve $(x, y) = (r\cos\theta, r\sin\theta)$.

b) A particle with mass $m$ and negative energy $E$ moves in the potential

$$V(r) = -\frac{k}{r} + \frac{\beta}{2r^2}.$$

Here, $k$ and $\beta$ are positive constants. Show that the orbit can be written in the form

$$r = \frac{p}{1 + \varepsilon \cos(\gamma\theta)},$$

i.e., a precessing elliptic orbit (cf curve in a)). Find expressions for $p$, $\varepsilon$ and $\gamma$. Show that the semimajor axis is $a = k/|E|$, the same as when $\gamma = 1$.

(This is a *model* potential that reproduces, qualitatively, the precession of planetary orbits, due to GR (general relativity). The correction in GR is actually proportional to $-1/r^3$.)

## Question 2

A spherical pendulum with a massless rod of length $d$ and mass $m$ moves in the gravitational field $\boldsymbol{g} = -g\hat{z}$. See figure on page 2. Use the regular azimuth angle $\phi$ and the polar angle $\theta$ between the rod and the *negative* $z$ axis in this problem. Let $V = 0$ for $\theta = \pi/2$.
Show that the Lagrange equations give

$$p_\phi \equiv md^2 \sin^2\theta\, \dot{\phi} = \text{constant}$$

and

$$\ddot{\theta} - \frac{1}{2}\sin 2\theta\, \dot{\phi}^2 + \frac{g}{d}\sin\theta = 0.$$

Next, show that the total energy $E$ can be written as

$$E = \frac{1}{2}md^2\dot{\theta}^2 + V_{\text{eff}}(\theta).$$

Finally, show that the time $t$ and the angle $\phi$ can be written in integral form as

$$t = \sqrt{\frac{md^2}{2}} \int \frac{d\theta}{\sqrt{E - V_{\text{eff}}(\theta)}}$$

and

$$\phi = \frac{p_\phi}{2md^2} \int \frac{d\theta}{\sin^2\theta\sqrt{E - V_{\text{eff}}(\theta)}}.$$

If $p_\phi = 0$, what type of motion is executed by the pendulum?

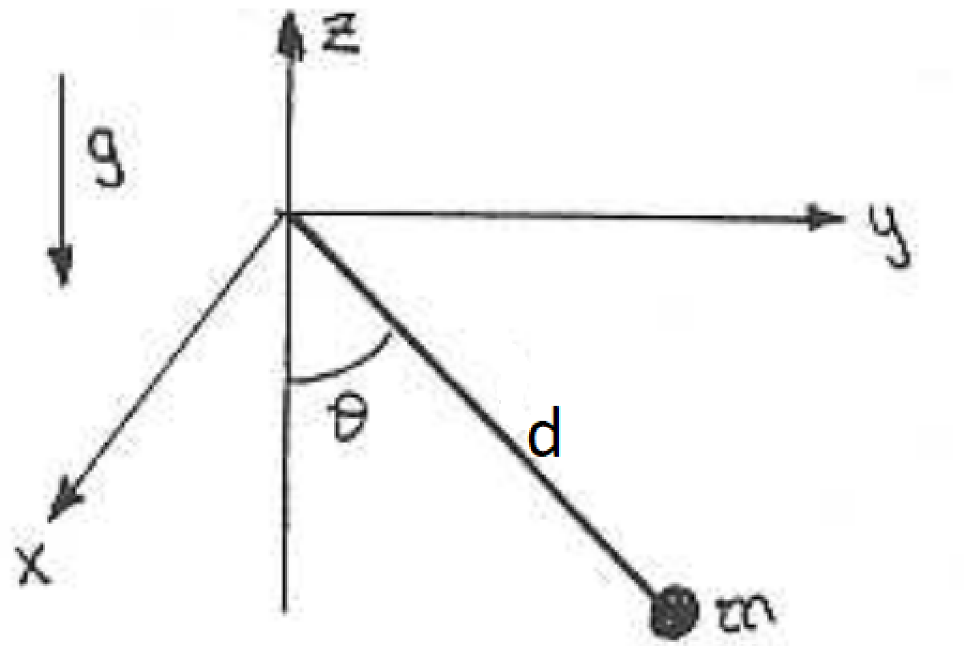


Figure 1: Spherical pendulum.