TFY4345 Classical Mechanics. Department of Physics, NTNU.

ASSIGNMENT 7

Question 1

Find the Lagrange equations and Hamilton's equations expressed in cylindrical coordinates $q_i = (r, \theta, z)$ for a particle with mass m in a potential $V = V(q_i)$.

Question 2

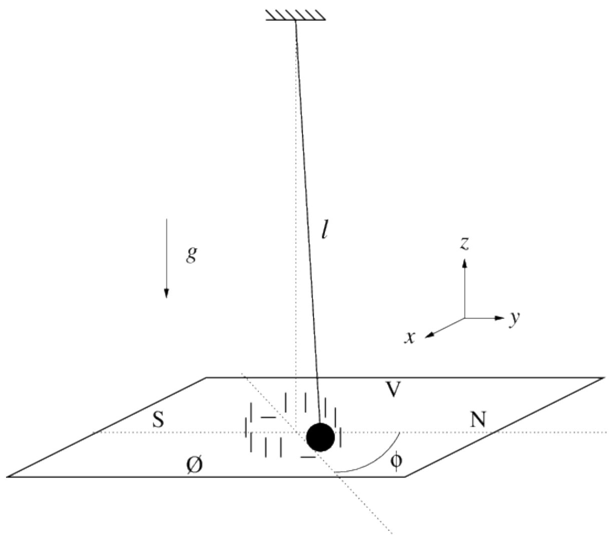

Figure 1: Foucault pendulum.

The Foucault pendulum in Realfagbygget swings back and forth in a plane that rotates with angular frequency

$$\dot{\phi} = \omega \sin\theta$$

because of the Coriolis force $2m\boldsymbol{v} \times \boldsymbol{\omega}$. Here, $\omega = 2\pi/24\ \mathrm{h}^{-1}$ is the angular velocity of the earth and θ is the latitude, ca $63^\circ\, 36'$ in Trondheim. In other words, the sticks on the floor are all kicked down by the pendulum within about 13.4 hours. In this exercise, you will derive the above result.

a) The pendulum wire has length ℓ and a sphere with mass m. We assume a massless wire and small oscillations. You may ignore the centrifugal force. The forces acting on m are therefore gravity, the force from the wire, and the Coriolis force. Choose a coordinate system with m in origo when the pendulum is in equilibrium. The (positive) x and y axes are directed towards east and north, respectively.
With these assumptions, show that the movement of m in the xy plane is governed by the differential equations

$$\begin{aligned} \ddot{x} - 2\Omega\dot{y} + \omega_0^2 x &= 0, \\ \ddot{y} + 2\Omega\dot{x} + \omega_0^2 y &= 0. \end{aligned}$$

Here, $\Omega = \omega \sin\theta$ and $\omega_0 = \sqrt{g/\ell}$.

b) Introduce the complex variable $u = x + iy$ and show that the two equations above may be combined into a single equation

$$\ddot{u} + 2i\Omega\dot{u} + \omega_0^2 u = 0.$$

c) Solve the equation for u with the ansatz $u = \exp(\alpha t)$. Use the initial conditions $x = y = \dot{y} = 0$ and $\dot{x} = v_0$ at $t = 0$. Show that m moves in the xy plane with

$$\begin{aligned} x(t) &= \frac{v_0}{\omega_0} \cos\Omega t \, \sin\omega_0 t, \\ y(t) &= -\frac{v_0}{\omega_0} \sin\Omega t \, \sin\omega_0 t. \end{aligned}$$

In other words, harmonic oscillations with angular frequency ω_0, where the pendulum plane rotates around the z axis with angular frequency Ω.